CALCULATING THE RATE OF DESCENT OF A HELICOPTER DURATION ROCKET

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Introduction

Over the past decade the race to improve the performance of the helicopter duration model has really taken off. There have been numerous NARAM R&D reports done on the subject, and countless rocket designs put forward. This paper will concentrate on the aerodynamics of the rotating model, because that is the area that seems to be the least known and understood, and it may lead to significant performance increases.

The sequence of a helicopter flight can be broken down to three phases: boost and coast, deployment of the rotors and transition to helicopter flight, and finally, full autorotation.

The boost and coast portion of a helicopter flight is very similar to any model rocket in flight, and much has been written on the subject (i.e.,*Topics in Advanced Model Rocketry* published by by MIT Press). Once the model has reached the end of its coast and when the ejection charge goes off (hopefully at the apogee of the trajectory), the rotors are deployed, and the model begins the helicopter portion of the flight.

The first step in defining the models flight is to look at its *free body diagram*. The free body diagram is a simplistic representation of all the forces acting on the model. For any free-falling object, there are two forces that act on it. These are gravity and aerodynamic drag. The aerodynamic drag is a frictional force that retards the downward motion of the falling object, and is directly proportional to the velocity of the object.

$$F_f \propto b \bullet v^n \tag{1}$$

Actually F_f is the drag force, and it is well know that drag is defined as:

$$D = 1/2 \bullet \rho \bullet C_D \bullet S \bullet v^2 \tag{2}$$

Therefore b is equal to $1/2 \bullet \rho \bullet C_D \bullet S$, and $n=2$. Then the aerodynamic frictional force is:

$$F_f = b \bullet v^2 \tag{3}$$

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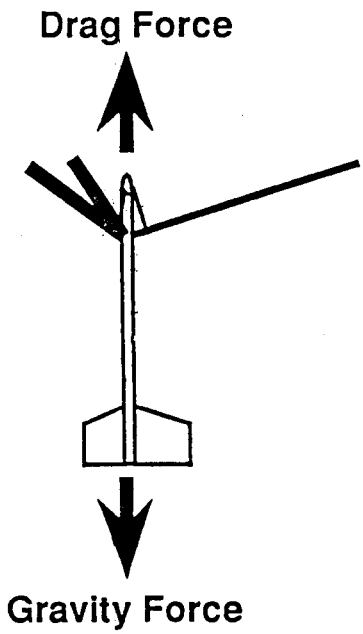

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Figure 1

Figure #1 shows the forces on the model. Summing the forces up gives the equation:

$$F = -m \cdot g - b \cdot v^2 \tag{4}$$

Notice that the drag force is negative. This is because it retards the velocity. The drag force will be upward when the velocity is down, and the force will be downward when the velocity is up. The gravity term is negative because it is acting in the negative (downward) direction.

From Newton's third law, the overall force is equal to the mass of the rocket times its acceleration.

$$m \cdot a = -m \cdot g - b \cdot v^2 \tag{5}$$

Dividing both sides by the helicopter mass yields:

$$a = -g - \left(\frac{b}{m}\right) \cdot v^2 \tag{6}$$

Because of the drag force on the model, the velocity will finally reach a maximum value. At this point the downward acceleration goes to zero.

$$0 = -g - \left(\frac{b}{m}\right) \cdot v^2 \tag{6a}$$

Solving for the maximum velocity (called *terminal velocity*) produces:

$$v = \sqrt{\frac{g \cdot m}{b}} \tag{7}$$

or:

$$v = \sqrt{\frac{2g \cdot m}{\rho \cdot C_D \cdot S}} \tag{7a}$$

The terminal velocity is the maximum falling speed of the auto-gyrating helicopter. The speed is dependent on three variables: the mass of the rocket, the reference area (S) chosen, and the drag coefficient of the falling model.

Obviously, the mass of the rocket should be kept to a minimum. The reference area will most likely be the total rotor planform area. If that is the case, the rotor area should be as large as possible. This makes sense, but has a associated weight penalty.

The coefficient of drag is one term that needs clarification. This is the drag coefficient of the falling helicopter. It is different from the drag coefficient of the rotor itself. This overall drag coefficient is not only dependent on the rotor drag coefficient, but also on the rotation rate of the helicopter, the lift produced by the rotation, and the total planform area of all the rotors. More will be said on this term later.

The time it takes to fall to reach terminal velocity is difficult to determine accurately because of the dynamic nature of the system during the transition phase. First the rocket has no rotation rate, so it begins to fall by itself, and the drag is a function of the planform area (frontal area). As the potential energy of the falling rocket is converted into kinetic energy and rotational energy, the drag coefficient changes. Since it is constantly changing, it becomes difficult to evaluate the time to terminal velocity.

The time to terminal velocity, then, must be estimated. This is started by reviewing equation number six. Acceleration is the time derivative of velocity, the left side of equation six can be replace by dv/dt.

$$\frac{dv}{dt} = -g - \frac{b}{m} \cdot v^2 \tag{8}$$

Separating variables and rearranging gives:

$$\frac{dv}{\left(g + \frac{b}{m} \cdot v^2\right)} = -dt \tag{9}$$

Both sides of equation nine are multiplied by b/m, and then both sides are integrated. The resulting expression is:

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$$\int \frac{dv}{\left(\frac{m \cdot g}{b} + v^2\right)} = \int -\left(\frac{b}{m}\right) \cdot dt \tag{10}$$

The left side is in the form of the equation solved thus:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \cdot \tan^{-1} \frac{x}{a} \tag{11}$$

When this rule is applied to equation nine, the result is:

$$\frac{1}{\sqrt{\frac{m \cdot g}{b}}} \cdot \tan^{-1}\left(\frac{v}{\sqrt{\frac{m \cdot g}{b}}}\right) = -\frac{b \cdot t}{m} + C_1 \tag{12}$$

The constant of integration, C_1, depends on the initial conditions. Choosing v=0 at t=0 (meaning that the rocket falls from apogee where the velocity is zero) yields:

$$\frac{1}{\sqrt{\frac{m \cdot g}{b}}} \cdot \tan^{-1}(0) = -\left(\frac{b \cdot (0)}{m}\right) + C_1 \tag{13}$$

Therefore, $C_1 = 0$ (14)

The equation 12 becomes:

$$\frac{1}{\sqrt{\frac{m \cdot g}{b}}} \cdot \tan^{-1}\left(\frac{v}{\sqrt{\frac{m \cdot g}{b}}}\right) = -\frac{b \cdot t}{m} \tag{15}$$

Rearranging gives:

$$\tan^{-1}\left(\frac{v}{\sqrt{\frac{m \cdot g}{b}}}\right) = -\sqrt{\frac{m \cdot g}{b}} \cdot \frac{b \cdot t}{m} \tag{16}$$

Taking the tangent of both sides gives:

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$$\left(\frac{v}{\sqrt{\frac{m \cdot g}{b}}}\right) = \tan\left(-\sqrt{\frac{m \cdot g}{b}} \cdot \frac{b \cdot t}{m}\right) \tag{17}$$

or:

$$\left(\frac{v}{\sqrt{\frac{m \cdot g}{b}}}\right) = -\tan\left(\sqrt{\frac{m \cdot g}{b}} \cdot \frac{b \cdot t}{m}\right) \tag{18}$$

Finally, separating velocity by itself yields:

$$v = -\left(\sqrt{\frac{m \cdot g}{b}}\right) \cdot \tan\left(\sqrt{\frac{m \cdot g}{b}} \cdot \frac{b \cdot t}{m}\right) \tag{19}$$

This last equation is the vertical velocity of the falling helicopter rocket. This is the velocity that for helicopter duration needs to be minimized. The first term shows that for the vertical velocity to be a minimum, the mass needs to be a minimum, and that the term "b" needs to be a maximum. From equation number two, the coefficient of drag needs to be maximized.

Rotor Blade Aerodynamics

Near the beginning of this paper, we found the basic equation of the models rate of descent. This equation was:

$$V = \sqrt{\frac{2 \cdot g \cdot m}{r \cdot C_d \cdot S}} \tag{7a}$$

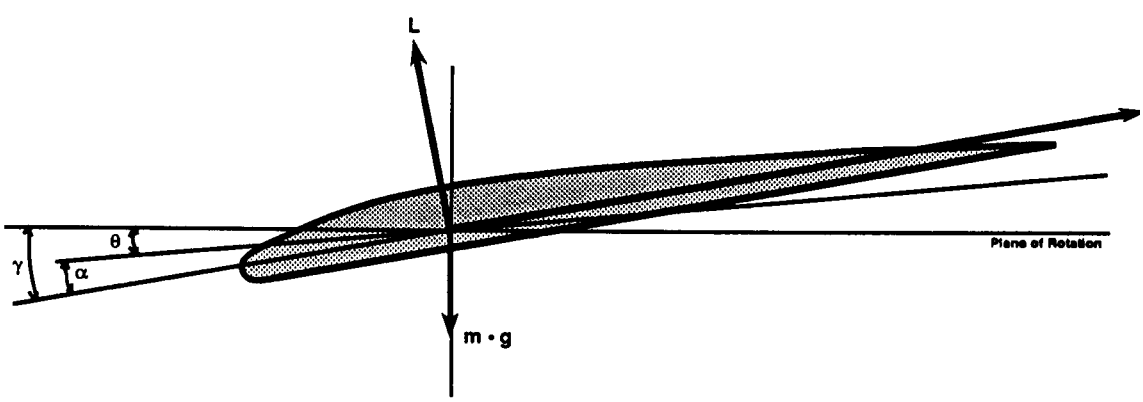

Figure 2

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The question that now must be answered is how is the overall helicopter drag coefficient (C_D) determined. To find that answer, one must look at the individual rotor blades themselves. Figure 2 defines the pertinent axes, angles and velocities used in the flight mechanics of the rotor:

γ = the flight path angle, negative for descending flight (glide)
α = rotor (reference) angle of attack
L = rotor lift
D = rotor drag
R = vector resultant force of "L" and "D"
W = model weight
θ = pitch attitude angle
V = true airspeed
$V_V = RD$ = rate of descent (vertical flight speed)

In the flight condition shown for stable flight:

$$\mathbf{R} + \mathbf{W} = 0 \tag{20}$$

Introducing a term:

$$C_R \cdot \mathbf{q} \cdot S = (C_L^2 + C_d^2)^{0.5} \cdot \mathbf{q} \cdot S = \mathbf{W} \tag{21a}$$

as the scalar equivalent of equation 20, where C_R is the resultant force coefficient, and **q** is:

$$\mathbf{q} = .5 \cdot r \cdot V^2 \tag{21b}$$

writing equation 20 into its component forms:

$$-D - W \cdot \sin\gamma = 0 \tag{22a}$$
$$L - W \cdot \cos\gamma = 0 \tag{22b}$$

are obtained. The glide-path angle is:

$$V_V = V \cdot \sin\gamma = RD$$

Equations 22a and 22b can be rewritten as:

$$C_d \cdot \mathbf{q} \cdot S = W \cdot \sin\gamma \tag{23a}$$

$$C_L \cdot q \cdot S = W \cdot \cos\gamma \tag{23b}$$

Note that the variables thus far are W (mass • gravity), ρ (a function of altitude), α, V and γ. Two of the variables will be considered fixed for a particular model; these are W, and ρ.

It is generally most convenient to express the flight mechanics quantities: glide angle, airspeed, and rate-of-descent in the aerodynamic coefficients C_L and C_d.

By dividing equation 23a by 23b, it follows that:

$$\tan\gamma = \frac{C_d}{C_L} \tag{24}$$

From equations (21a) and (24) it can be found that:

$$V = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_R}} \tag{25}$$

and / or:

$$V = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L} \cdot \cos\gamma} \tag{26}$$

The rate-of-descent follows from equations (24) and (26) as:

$$RD = V \cdot \sin\gamma = V \cdot \frac{C_d}{C_L} \cdot \cos\gamma = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{C_d^{\,2}}{C_L^{\,3}} \cdot \cos^3\gamma} \tag{27}$$

If the glide path angle (γ) is so small that $\cos\gamma \simeq 1$ is satisfied (a shallow glide), then:

$$V = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{1}{C_L}} \tag{28}$$

$$RD = V \cdot \frac{C_d}{C_L} = \sqrt{\frac{W}{S} \cdot \frac{2}{\rho} \cdot \frac{C_d^{\,2}}{C_L^{\,3}}} \tag{29}$$

It can be seen from equations (24) and (29) that the aerodynamic ratios C_L/C_d and C_L^3/C_d^2 play an important role in determining the rate-of-descent of the helicopter model.

From equation (29) it can be seen that the minimum rate of descent occurs when C_L^3/C_d^2 is a maximum. It must be remembered that C_L and C_d depend on the angle of attack, so these must be read off a chart for each value of α. Additionally, the velocity in equation (29) is a reference velocity, because for a helicopter, the tips travel at a higher velocity than the roots. The second part of equation (29) is valid because the C_L and C_d do not depend upon the forward velocity at low Mach numbers ($M \ll .3$).

Notice from equation (29) that the rate of descent is a direct function of C_L and C_d and therefore, the airfoil that is chosen. Once the airfoil is chosen, the best angle of attack would be found (by tabulating the RD versus α).

For a constant angle of attack wing, the table below summarizes the parameters that would have to be calculated and the order that they would be found.

α	C_L	C_d	$\frac{C_L^3}{C_d^2}$	RD

If there was interest in knowing the overall drag (C_D) of a model, it could be solved for in equation (7a) by setting the rate-of-descent equal to the terminal falling velocity.

Rotor Twist

To summarize, the rate of descent is a minimum when C_L^3/C_d^2 is a maximum. This occurs at a specific angle of attack (α) of the rotor into the airflow. So therefore, the rate of descent depends on the rotors being set at the optimum angle into the airstream for maximum rotor efficiency.

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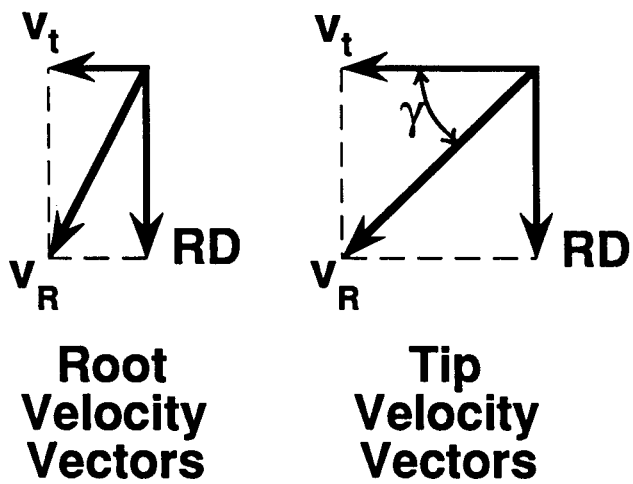

Figure 3

From Figure 3, you can see that the angle α changes over the span of the wing. This is because the forward velocity component of the rotor blade changes across the span of the wing (i.e. the speed of the rotor blade tip is faster than the speed at the root of the rotor). This is given from the equation:

$$v_t = \omega \cdot r \tag{30}$$

where "v_t" is the tangential velocity of the blade, "ω" is the rotational rate of the rotor with units of radians/sec, and "r" is the radius of the disk created by the spinning rotor (measured outward from the center of the rocket).

The fact that the angle a changes over the length of the wing leaves us with a dilemma. On a straight rotor blade, maximum efficiency will not be achieved because most of the blade will be at the wrong angle of attack to a maximum C_L^3/C_d^2.

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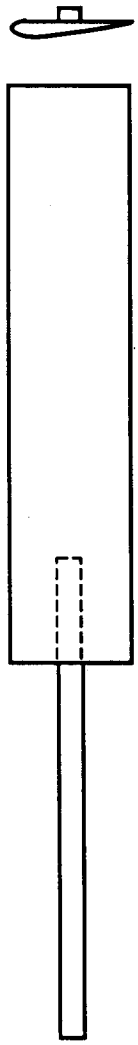

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Figure 4

The first solution to this problem is to remove the inner part of the rotor blade, and replace it with a shaft extending from the hub to support a short wing (see figure 4). George Gassaway has built this type of model, and you can see a picture of it on page 3 of the September-October 1989 issue of *Space Coast Rocketry*.

By creating this unusual type of model, you are trying to eliminate the problem completely by removing the inner portion of the blade that is the least effective anyway at producing lift (because the slower velocity the rotor is traveling at near the hub). There is one problem with this; the shaft supporting the blades produces drag which tends to counteract the rotation of the model. To alleviate some of this drag, you could shape the shaft like a symmetrical airfoil. But even a symmetrical airfoil at the wrong angle will generate

lift (and drag), and this leads us back to having a chambered airfoil not aligned at the most efficient angle.

The second way of solving the incorrect angle of attack problem is to twist the blade, so as to adjust the blade for maximum efficiency for the chosen airfoil. This needs to be designed into the blade from the start, so let's take a look as to how this works.

At this point, the airfoil has been selected, and the optimal angle-of-attack (α) has been determined. The angle-of-twist (θ) can be found by using figure 2 and a little bit of trigonometry.

The angle γ, which the angle-of-attack plus the angle-of-twist can be found from the equation:

$$\tan g = \frac{RD}{\omega \cdot r} \tag{31}$$

$$\tan(\theta + \alpha) = \frac{RD}{\omega \cdot r} \tag{32}$$

Taking the tangent of both sides yields:

$$(\theta + \alpha) = \tan^{-1}\left[\frac{RD}{\omega \cdot r}\right] \tag{33}$$

Or finally the angle-of-twist is:

$$\theta = \tan^{-1}\left[\frac{RD}{\omega \cdot r}\right] - \alpha \tag{34}$$

How do you use this twist angle? Well, you need to design it into your wing by choosing the rotation rate (ω) that you want your model to spin at. A good estimate to start at is 3 revolutions per second or 6π rad/sec (where π has a value of 3.14). From this point you calculate θ for different portions of the wing along the span and then build in twist to match the angles you've calculated.

You should find that the twist will be greater at the root than at the tips of the rotor blades.

A third way to solve the angle-of-attack problem is to actually change the airfoil shape along the span of the wing. Physically it can be done, but it must be done gradually, and the selection of airfoils can make it difficult to do. Each region of the airfoil has a different angle-of-attack, and a different velocity, so there are many variables to consider.

If you have built one of the original versions of the Rotaroc, you have been changing the airfoil (maybe unknowingly) by cutting it diagonally spanwise like almost all the plans show (see figure 5).

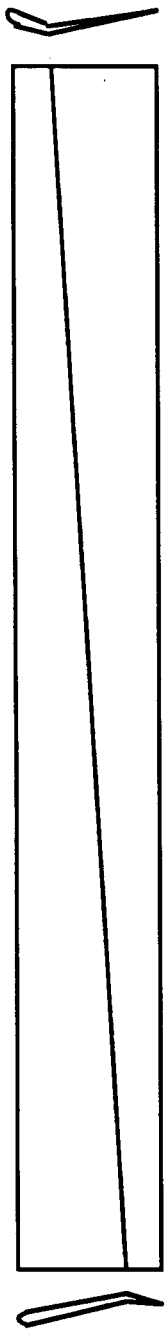

Figure 5

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There is a bit of uncertainty of the aerodynamics of that particular airfoil, and it would take wind tunnel experimentation to fully understand the con-

sequences (or benefits) of such a wing shape, so I won't go into to much detail here.

The method for calculating the rate of descent for such a wing can be broken into sections. To keep the rate-of-descent equation simple, you should try to select airfoil sections that will yield the same C_L^3/C_D^2. This is difficult because of the different angle-of-attacks you will have to deal with.

If you try this method, it is theoretically possible to keep the wing straight without having to use any twist. It may be to only way to use solve the angle-of-attack problem on Rose-A-Roc type models. I'd be interested in any such solution if you come up with one.

A Example Showing How To Use The Equations

In order to bring some cohesion to the subject, in this issue we'll go through an example on how the rate-of-descent is determined and maybe shed some light on things that can be changed to improve the performance even more.

As derived in the last article, the total performance depends largely on the airfoil shape selected. For the example here, we will choose an airfoil that is very popular among rocketeers, the "Clark Y." This is probably one of the simplest to make because it has a flat bottom, which is easy to sand. Figure 6 shows the airfoil characteristics for this particular airfoil (although the Reynolds numbers is 3,700,000, for this exersize the numbers will have to suffice).

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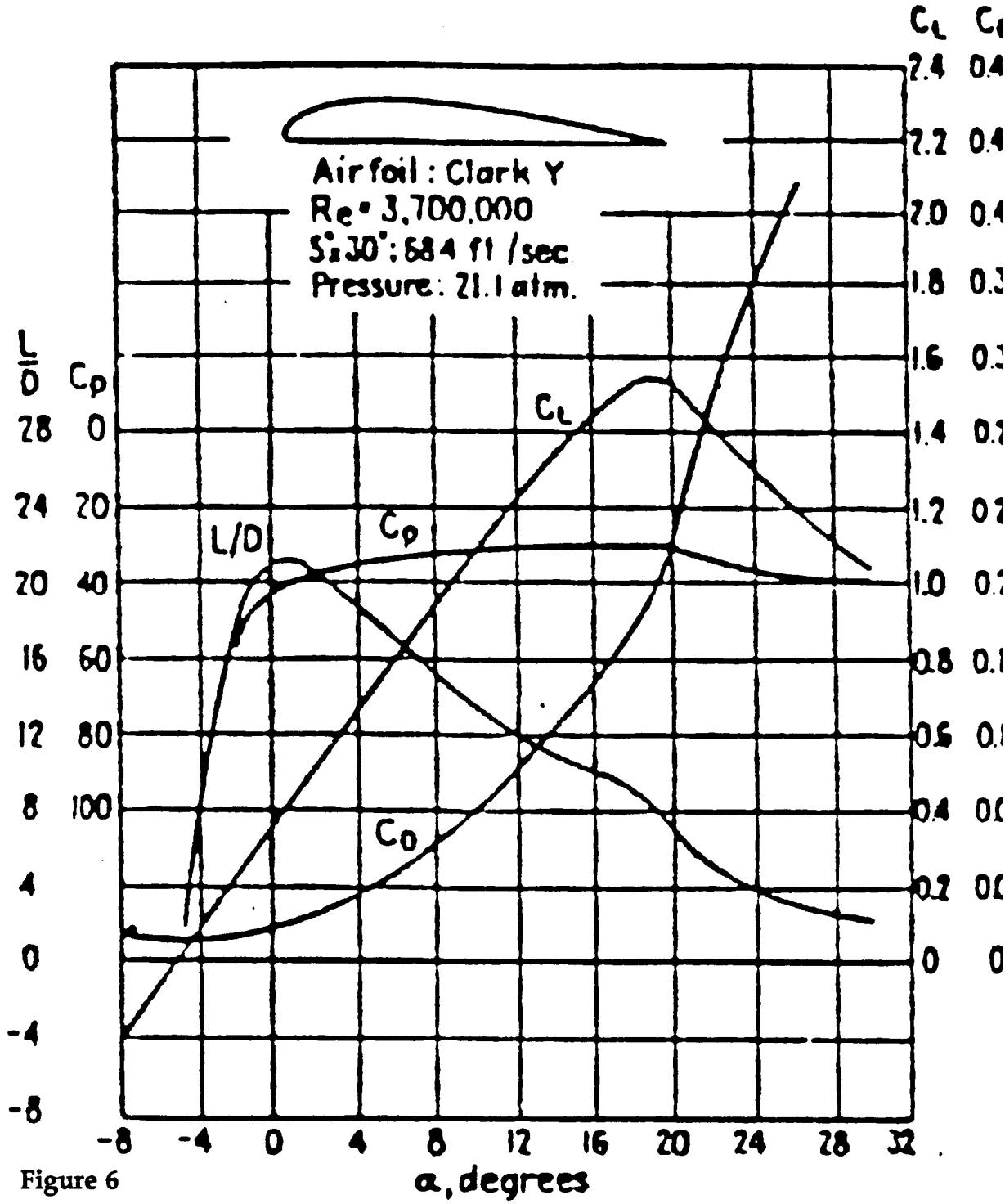

Figure 6

The mass of our hypothetical rocket will be 0.255 Kilograms (9 oz.). We'll choose a rotor with a span of 0.38 meters for a total disk diameter of 0.762 meters (30 inches). Next we'll get into the rotation aerodynamics.
The rate at which we'll design our helicopter to spin at will be 3 revolutions-per-second. Converting:

$$\text{Angular Velocity} = \omega = 3\,\frac{\text{rev}}{\text{sec}}$$

$$\omega = \left(3\,\frac{\text{rev}}{\text{sec}}\right)\left(\frac{360°}{\text{rev}}\right)\left(\frac{2\pi\ \text{rad}}{360°}\right)$$

$$\omega = 6\pi\ \left(\frac{\text{rad}}{\text{sec}}\right)$$

The velocity that the tip of the rotor blades is found by the equation:

$$v_{tip} = \omega \times r$$

$$v_{tip} = 6\pi\left(\frac{\text{rad}}{\text{sec}}\right) \times 0.38\ (\text{m})$$

$$v_{tip} = 7.163\left(\frac{\text{m}}{\text{sec}}\right)$$

The velocity at the root will be taken 0.01 meters from the center of the rocket. This is because the hinge is mounted on the body tube of the rocket, and that is the distance to the outside of the body tube. So the velocity at the root of the blade is:

$$v_{root} = \omega \times r$$

$$v_{root} = 6\pi\left(\frac{\text{rad}}{\text{sec}}\right) \times 0.01\ (\text{m})$$

$$v_{root} = 0.188\left(\frac{\text{m}}{\text{sec}}\right)$$

This is a good point in which to pause and reflect on the assumptions made up to this point. Thus far, there have been two assumptions. The first has been that the helicopter will rotate at 3 revolutions per second. This number was taken from the NARAM 25 R&D report by Matt Steele, George Gassaway and Patrick McCarthy (the Zunofark Team) entitled *Rotor Twist on Rotoroc Models* of measurements they took. So it is a realistic assumption. The other assumption is that the data for the "Clark Y" airfoil is applicable to the situation at hand. I said before that it wasn't; but how do you check this for yourself and for any airfoil. The data for the airfoil was taken at a Reynold's number of 3,700,000, so this is a good place to make the check.
The Reynold's number is defined as:

$$RN = \frac{\rho \times v \times L}{\mu}$$

where "ρ" is the density of the air, "v" is the velocity of the air over the airfoil (or of the airfoil moving through the air), "μ" is the "Coefficient of Viscosity,"

and "L" is a reference length (usually taken as the length of the chord of the airfoil from the leading edge to the trailing edge.) We just found two velocities (tip velocity and root velocity), so we can find the Reynold's numbers at those two important places if we can find the values of the other variables in the equation.

The density and coefficient of viscosity of air will be taken at sea level, since most of Florida (where I write this), is approximately at that altitude. The altitude at which you launch your rocket may be different, and you may have to compensate for this. From the U.S. Standard Atmosphere Tables, the values of ρ and μ at sea level are:

$$\rho = 1.2250 \frac{kg}{m^3}$$

$$\mu = 1.798 \times 10^{-5} \frac{kg}{sec \times m}$$

The length of the airfoil chord is given by design. In this example, the chord length will be:

$$L = 0.0381 \text{ m} \; (= 1.5 \text{ inches})$$

With all this known, the Reynold's number at the tip of the rotorblade will be:

$$RN_{tip} = \frac{1.2250 \left(\frac{kg}{m^3}\right) \times 7.163 \left(\frac{m}{sec}\right) \times 0.0381 \text{ (m)}}{1.798 \times 10^{-5} \left(\frac{kg}{sec \times m}\right)}$$

$$RN_{tip} = 18{,}593.72$$

Similarly, the Reynold's number at the root of the rotor is:

$$RN_{root} = \frac{1.2250 \left(\frac{kg}{m^3}\right) \times 0.188 \left(\frac{m}{sec}\right) \times 0.0381 \text{ (m)}}{1.798 \times 10^{-5} \left(\frac{kg}{sec \times m}\right)}$$

$$RN_{tip} = 306.3$$

Both the Reynolds number's found are obviously to low for the airfoil data to be applicable. So the results of the descent speed and the rotor twist

angle of the falling helicopter model will be erroneous. The solution to this problem would be to find airfoil data that would be applicable to the Reynolds number regimes that we are using (a good source is a book written for model airplane designers entitled "*Model Airplane Aerodynamics*" (available from Zenith Books 1-800-826-6600 for the price of $29.95 product number 111933AE). But we will continue the example just to show how the results can be found given the correct data.

From Figure 6 we will pull data points off the graph to find the most efficient configuration of the rotor blade. From the second article in this series (Rotor Blade Aerodynamics) we said that the blade is most efficient when C_L^3/C_d^2 is at the maximum value. The graph doesn't plot this, so we will pick-off the C_L and the C_d for a given angle of attack (α) and then simply calculate the value of C_L^3/C_d^2. The table below summarizes the results.

α	C_L	C_d	C_L^3/C_d^2
-4	0.10	0.015	4.444
0	0.35	0.020	107.188
4	0.65	0.038	190.184
8	0.95	0.058	254.860
12	1.20	0.090	213.333
16	1.49	0.140	168.770
20	1.62	0.200	106.280
24	1.55	0.300	41.370

From the table we can see that the maximum value of C_L^3/C_d^2 occurs when the airfoil is at a 8° angle of attack relative to the airflow. The minimum rate of descent can now be determined from equation 29, also from that second article on helicopter theory:

$$RD = \sqrt{\frac{W}{S} \times \frac{2}{\rho} \times \frac{C_d^2}{C_L^3}}$$

The variable "S" is the total wing area which is found multiplying the length of each rotor by the chord length and then multiplying that by the number of rotors. In our example the area is:

$$S = 0.0423\ m^2$$

Then the rate of descent becomes:

$$RD = \sqrt{\frac{\left(9.8\left(\frac{m}{sec^2}\right) \times 0.255\ kg\right)}{0.0423\ m^2} \times \frac{2}{1.225\left(\frac{kg}{m^3}\right)} \times \frac{1}{254.86}}$$

$$RD = 0.6151 \frac{m}{sec}$$

How does this compare with real helicopter duration models? Again, going back to the NARAM 25 R&D report of the Zunofark Team, we would find this number a lot lower than actual descent velocities of real models. They found that a model with an 18 inch (0.46 m) rotor length and a chord of 1.5 inch (0.038 m) of the same airfoil type, fell at a rate of 1.37 m/sec.

Why does our example model fall at less than half the rate of the "improved helicopter model" (by improved, they meant it had a twisted airfoil)? There are a couple of reasons that this might be. First as we said twice before, the airfoil data we relied on is not totally applicable to our "real world" model. Because of the higher Reynold's number data, we are expecting the wing to produce more lift than it is capable of during actual flight. The second reason may be because the airfoil used by the Zunofark team was not twisted at the most optimum angle, thereby increasing the descent speed of their model. In our example, we have assumed that the air is flowing over the airfoil in the most efficient (hence optimum) angle.

The next question you'll have is how do you get the air flowing over the wing at the optimum angle? Since we already know the optimum angle-of-attack (α). We simply need to find the angle θ that we need to twist the wing to for the optimum angle (see figure 2).

The method used to find the angle θ was derived in equation 34 which is:

$$\theta = \tan^{-1}\left[\frac{RD}{\omega \times r}\right] - \alpha \tag{34}$$

With this equation, you calculate the angle θ at the tips of the rotors and at the root, and then twist the rotor to achieve this result. To continue with our example, the angle θ at the root would be:

$$\theta = \tan^{-1}\left[\frac{.6151}{6\pi \times 0.01}\right] - 8°$$

$$\theta = 64.97°$$

At the tip of the blade, θ is:

$$\theta = \tan^{-1}\left[\frac{.6151}{6\pi \times 0.38}\right] - 8°$$

$$\theta = -3.09°$$

This report was downloaded from the Apogee Components web site: www.ApogeeRockets.com

The negative means that the tip will have a positive angle-of-attack because we took downward to be the positive direction.

At this point, the model is "designed" and all that remains is to build it. The actual twisting of the rotor can be accomplished as descrived in the Zunofark team R&D paper. In it they describe these basic steps: 1) sand the blade into the correct airfoil shape. 2) construct a jig to hold down the rotor to the correct angles. 3) Soak the rotor in a solution of water and liquid ammonia (the ammonia softens the ligin, or "glue" which bonds the wood fibers together) 4) clamp the now flexible rotor into the jig and allow to dry for several days.

Once the rotors are twisted, you'll have to attach them to your model. This you may find difficult because the twisted blades do not lie flat against the body tube as flat rotors do on a regular *Rotoroc*. The test model built by the Zunofark team for their R&D report made room for the twisted blades by changing the bodytube diameter — going from a BT-20 tube at the nose, to a BT-5 for the rotor storage area, and then back to a BT-20 for the tail and motor unit.

There are other ways to create twisted blades. One would be to start with a block of wood and carve out the rotor. As you might imagine, this would be very difficult and tedious to achieve a good airfoil. Another way would be to cut the blades out of foam using a hot-wire foam cutter. This would then require you to stiffen the blades with composite materials. This second method is the one I predict will be the future of winning helicopter rotor blades.

To sum up, The method used to design a rotor would be: 1) select a airfoil that will give a good blend between ease of construction and aerodynamics. Remember, you want an airfoil with a high C_L^3/C_d^2. 2) Select the size of the airfoil - the length and the chord width of the blade, and choose a speed you want the helicopter to rotate at. 3) Calculate the velocity of the tip and the root of the blade. 4) Make a check of the validity of the airfoil data by calculating the Reynold's number. 5) Find the optimum angle of attack by finding the maximum value of C_L^3/C_d^2. 6) You can calculate the rate-of-descent so that you get an idea on the performance of the model. 7) Find the angle of twist of the model. 8) Build the model.

How This Report Differs from Previous Works.

The knowledgable reader will most likely be aware of other reports written on the subject of helicopter duration. There were two major ones written and presented at previous NARAMS, and both were written by the team of George Gassaway and Matt Steele. The first was presented in 1981 at NARAM-23 (title unknown), and the second in 1985 at NARAM-25 which was entitled "Rotor Twist on Rotaroc Models."

My work presented in this paper differs basically in one major way — it derives the necessary equations of helicopter performance, and then leads to a method which any modeler can use to design a optimum performing helicopter model. The methods used to derive the equations are all stated as plainly as possible so that future reports written on this subject can be build upon justified methods and variables.

I used the two reports written by the *Zunofark Team* as a starting point for this paper. While studying the first on, I found a number of mathematical errors, and some unfounded assumptions that unfortuneately lead to the wrong conclusions. But it was using this report that I did find some valuable ideas, and I credit the authors for making a good attempt at trying to model the various forces acting on a helicopter duration model.

The NARAM-25 report by the same authors was much closer to what I believe is the correct model of helicopters in flight. My report differs from this one in a couple of significant ways. First, in their report they found the twist angle of the rotors by first assuming a descent velocity. The method that I've presented in this report ***finds*** the optimum rate of descent first, and then shows how to calculate the correct twist angle of the rotors to achieve that descent rate.

Second, the entire derivation of my paper hinges upon the fact that it is an airfoil that generates the lift and drag which allows the model to rotate and descend slowly in the air. Although this seams to be an obvious fact, I haven't found any previous works that even mention this premise. This is important, because it allows the modeler a method of choice in optimizing his model for his particular situation by choosing the right airfoil that generates the proper lift-to-drag ratio to suit the conditions.

By no means is the subject of helicopter duration exhausted. There are plenty of subjects that still need to be explored, and it is my hope that this report may provide a basis from where a modeler can start.

Cost

This research paper had no cost except for the cost of my time to research and formulate the ideas contained herein.

List of Terms:

F_f : Force of aerodynamic friction
D : Aerodynamic Drag
ρ : Density of Air
C_d : Coefficient of Drag (of rotor blade)
C_D : Coefficient of Drag (of descending model)
C_L : Coefficient of Lift
C_R : Resultant force coefficient
S : A reference area
V : True velocity
$v_v = RD$: Rate of descent (vertical flight speed)
v_t : Tangential velocity of blade
V_R : Resultant velocity vector
b : $1/2 \cdot \rho \cdot C_d \cdot S$
g : Acceleration due to gravity
F : Total of all Forces acting the model
m : Mass of the model
a : Acceleration
dv : Differential of velocity (a very small change in velocity)
dt : Differential of Time
α : Rotor (reference angle of attack)
L : Rotor lift (also used as *length* when calulating Reynolds Number)
R : Vector resultant force of "L" and "D"
W : Model weight
θ : Pitch attitude angle
M : Mach number
r : Radius of rotor as measured from hub
q : Dynamic pressure = $1/2 \cdot \rho \cdot V^2$
μ : Coefficient of Viscosity
RN : Reynolds number

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