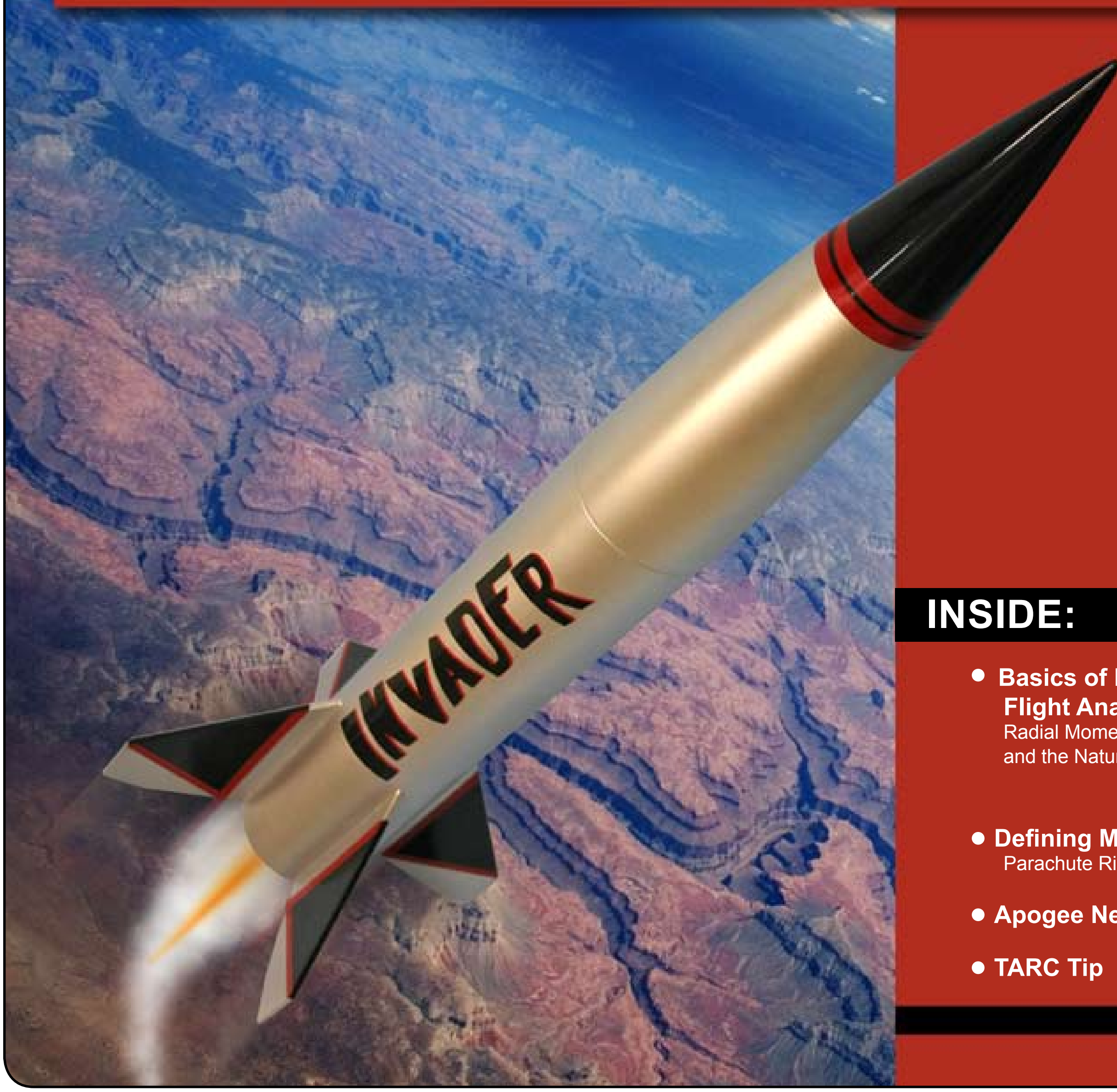
ISSUE 196 - November 06, 2007

APOGEE PEAK OF FLIGHT NEWSLETTER

BASICS OF DYNAMIC FLIGHT ANALYSIS

PART 4

Radial Moment of Inertia and the Natural Frequency



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PEAK OF FLIGHT

Basics of Dynamic Flight Analysis Part 4

Radial Moment of Inertia and the Natural Frequency

By Tim Van Milligan

This is the fourth part in the series of articles on Dynamic Flight Analysis. If you are new to the newsletter, the first three parts were covered in Newsletters 192, 193, and 195.

According to the book *Topics in Advanced Model Rocketry*, there are five parameters of fundamental importance to a rocket's dynamic characteristics. They are:

I_L = Longitudinal Moment of Inertia
C_1 = Corrective Moment Coefficient
C_2 = Damping Moment Coefficient
I_R = Radial Moment of Inertia
ω_n = Natural Frequency

In the previous articles of this series, we discussed the first three parameters: I_L, C_1, and C_2. While it all sounds very complicated, the use of the RockSim software allows us to easily see the affects when varying any of these design parameters. If you don't have RockSim, I urge you to get it today so you can see what a truly remarkable tool it is to optimize your rocket designs. It also makes this complex discussion of flight dynamics a lot easier to understand because it creates wonderful and simple-to-understand pictures.

Get RockSim today at: http://www.ApogeeRockets.com/rocksim.asp

Radial Moment of Inertia

That brings us to the Radial Moment of Inertia, I_R. It is very similar to the Longitudinal Moment of Inertia, the difference being that it is the measure of the amount of resistance (inertia) the rocket has about the "roll" axis of the rocket. The Longitudinal Moment of Inertia was the measure of resistance about the pitch and yaw axes of the rocket.

The Radial Moment of Inertia measures how much the rocket is going to resist rolling. The greater the number, the harder it will be to spin the rocket. You can accomplish this by designing fins with a very large span, then attaching weights to the tips. This configuration might be useful on a camera-carrying rocket where you want really rock-solid stability to keep the camera pointed in one direction. But other than that, increasing the Radial Moment of Inertia in this way only makes the rocket heavier and therefore reduces its performance (speed and altitude are diminished).

On the other hand, reducing the Radial Moment of Inertia makes it really easy to start a rocket spinning. And spinning rockets are really cool. It really helps decrease dispersion of the rocket, making it fly straighter. But if we say that we're going to ignore spinning rockets – which we are for this series of articles – then we can ignore the Radial Moment of Inertia.

I do want to point out to you that the RockSim software does calculate the Radial Moment of Inertia of each rocket design. And you can plot it out after running a simulation. The only thing you might want to do with it is to compare it to the Longitudinal Moment of Inertia. When you do, you'll see that the Radial Moment of Inertia is very tiny by comparison (see Figure 1).

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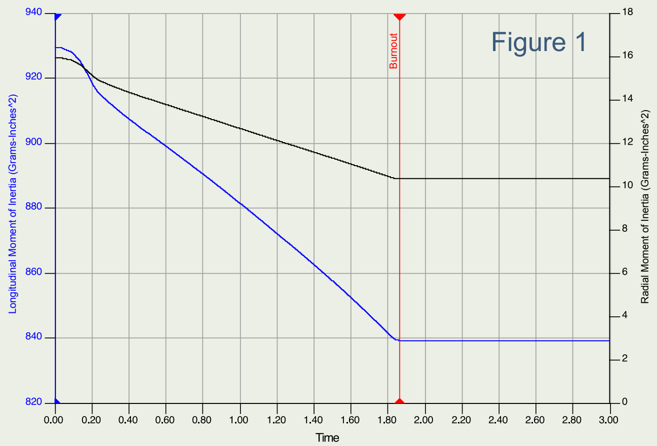

Figure 1: A rocket's Radial Moment of Inertia is tiny in comparison to its Longitudinal Moment of Inertia. This makes it easier to start it rotating about the roll axis compared to the pitch/yaw axis.

You'll also note from Figure 1 that the Radial Moment of Inertia decreases while the rocket engine is burning. That is because it is losing mass. When the mass of the rocket is constant, the Radial Moment of Inertia will be a constant value too.

But unless you have RS-PRO and are concerned about spinning rockets, you needn't concern yourself with the Radial Moment of Inertia. So this will end that discussion.

Natural Frequency

To begin our discussion of Natural Frequency, I want you to recall what happens to a rocket when it encounters a disturbance. Remember in the previous articles that we said that when a rocket encounters a disturbance in flight (such as a gust of wind), it starts to oscillate back and forth. Well, the Natural Frequency dictates how fast or slow these oscillations occur.

A rocket with a high Natural Frequency will oscillate back-and-forth faster than a rocket with a lower Natural Frequency.

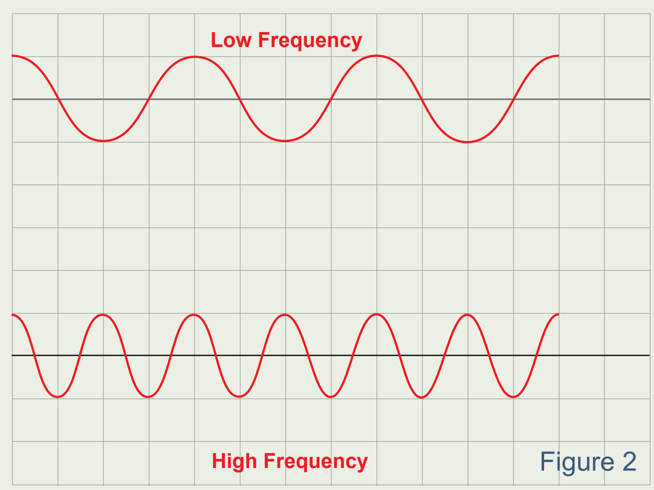

Figure 2: The Natural Frequency determines how fast a rocket will oscillate back-and-forth.

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The formula for finding a rocket's Natural Frequency is given by:

$$\omega_n = \sqrt{\frac{C_1}{I_L}}$$

Units: rad/sec
ω_n = Natural Frequency
C_1 = Corrective Moment Coefficient
I_L = Longitudinal Moment of Inertia

While the Longitudinal Moment of Inertia is not dependant on the rocket's velocity, the Corrective Moment Coefficient is. Therefore, the Natural Frequency of the rocket is also velocity dependant.

In other words, the faster the rocket flies, the higher the value for the Natural Frequency, and the rocket will oscillate back-and-forth faster. This can be seen in Figures 3 and 4.

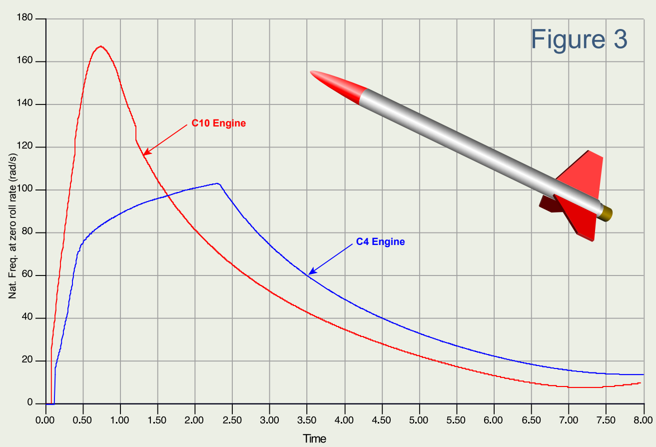

Figure 3: Because the C10 makes the rocket fly faster, it creates a higher Natural Frequency. Note also that the Natural Frequency curve is a similar shape as the velocity curve of the rocket.

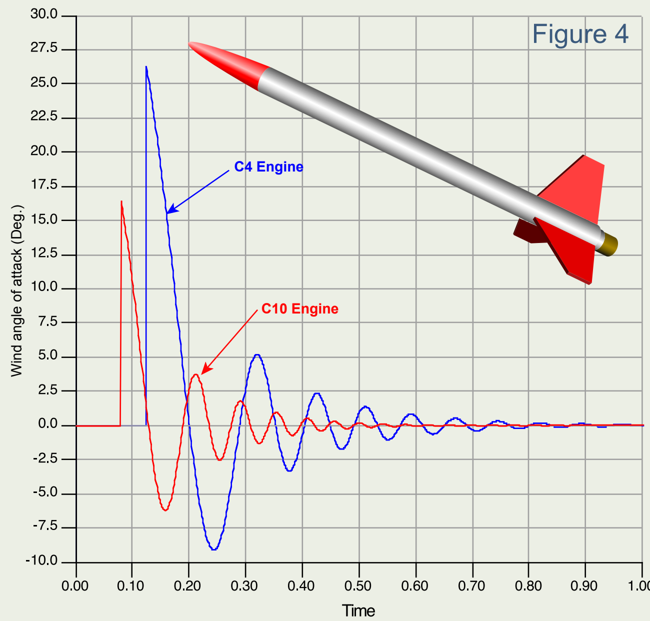

Figure 4: You can tell by looking at the AOA curve of this rocket that the rocket engine that makes the rocket fly faster (the C10) must have a higher Natural Frequency. The peaks of the oscillations of the red line are closer together.

You can also go back to our discussion on the Corrective Moment Coefficient and see that larger fins will also increase the Natural Frequency of the rocket. This is because larger fins increase the Corrective Moment Coefficient. In Chart 3 shown in Newsletter #193 http://www.ApogeeRockets.com/education/downloads/Newsletter193.pdf, you'll see that the oscillations are closer together for the rocket with the larger fins. That means it has a higher frequency of oscillation.

Increasing the Longitudinal Moment of Inertia will lower the Natural Frequency of the rocket (See Figure 3 in Newsletter #192 http://www.ApogeeRockets.com/education/downloads/Newsletter192.pdf). That chart shows that the oscillations take longer – the peaks are further apart – with a rocket that has a higher Longitudinal Moment of Inertia.

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Design Optimization Criterion for Natural Frequency

We've now reached the point in our discussion on dynamics that we can give the first generalized design procedures for an optimized rocket. By optimized, it is meant that the rocket is to be optimized to achieve maximum altitude.

Design Criterion: From the book *Topics in Advanced Model Rocketry* (page 610), it is recommended that the most desirable natural frequencies for *model rocket-sized* vehicles lie between 0.2V and 1.0V (where "V" is the velocity of the rocket in m/s).

According to the book, when the Natural Frequency is closer to 0.2V, that means "the Longitudinal Moment of Inertia is too great for the Corrective Moment Coefficient, so that the model cannot respond to the moments applied by its fins with sufficient rapidity for safe and stable flight."

The book goes on to say that "a higher value of Natural Frequency means the Longitudinal Moment of Inertia is too low, so the model is easily disturbed, or the corrective moment is too large, usually indicating too-large fins that cause excessive drag."

Getting The Natural Frequency Information From RockSim

RockSim can plot the Natural Frequency of a rocket (see Figure 3 above), but it is not in the format of Natural Frequency divided by the Velocity. Fortunately, it is very easy to get it from the software, so that you can check your design to see if it is fully optimized. Here are the steps:

1. Make sure you set the velocity of the rocket in the "Preferences" to the units of meter/sec.

2. Load your rocket design and run a simulation.

3. Click the "Plot Graph" button on the top of the main screen. In the graph preferences, change the x-axis to display "Velocity", and the y-axis to "Nat. Freq. At Zero Roll Rate (rad/s)."

4. Before clicking the plot button, you'll need to change the graph data starting and ending points. The graph should start at "When final engine burnout occurs," and it should end "When the rocket reaches apogee." The reason for this is to capture the linear portion of the curve. Since the Longitudinal Moment of Inertia is changing during the thrust phase, the graph during that portion of the simulation is non-linear.

5. The chart displayed will look like Figure 5. Notice how straight the line is. This is awesome! (I just love playing with RockSim, because it never ceases to amaze me of what it is capable.)

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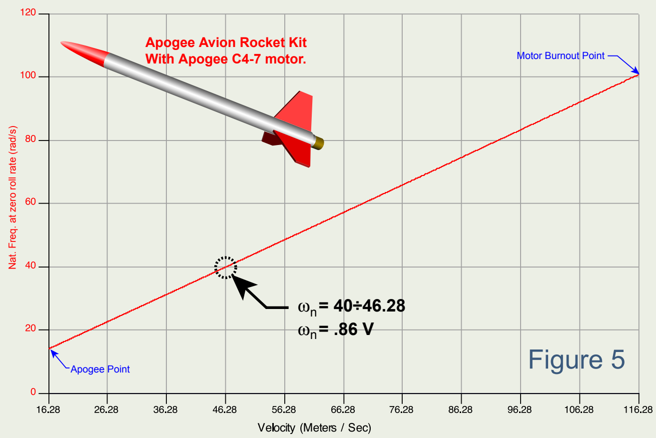

Figure 5: When you plot the Natural Frequency versus the velocity during the coasting phase of the flight, you'll get a straight line. This makes it easy to find the equation of the line.

6. Pick one point on the line and read off the Natural Frequency and the Velocity. Now simply divide the Natural Frequency by the Velocity at that point. This is the number you want to optimize. The number as mentioned above should be between 0.2 and 1.0.

That procedure isn't so bad, is it? I was able to get the information I needed in less than a minute. I'm sure you can do the same thing in no time at all.

In my example of the Apogee Avion rocket kit (*Coming Soon*) flown with a C4 motor, the Natural Frequency turned out to be 0.86 times the velocity. This is right in the range that we are looking for.

I want to point out one thing that I noticed while doing this. When I compared the graphs of the Apogee Avion kit with the C10 and the C4 motors, the curves were nearly identical. The reason is that the two engines have approximately the same burnout weight, so the Longitudinal Moment of Inertia for the two simulations will be very close. This tells me that it probably won't be hard to take a design and optimize it for similar size motors.

What about High Power Rockets?

Topics in Advanced Model Rocketry gives the Natural Frequency range for *model rocket sized vehicles* of 0.2 and 1.0 times the velocity (m/s). But since the book was written before the birth of high-power rocketry, it doesn't give a tolerance range for bigger rockets.

I did a check by looking at the Natural Frequency of a number of high-power rocket kits in RockSim. They were in the range of .15 to .18 times the velocity of the rocket in m/s. I would have thought this was too low by the criteria in the book, but these are proven designs that are being flow successfully by a lot of hobbyists. My guess is that these lower values are still safe, but the rocket is probably not optimized for peak altitude.

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There could be some research done in this area to find out what values are sufficient for the Natural Frequency of big rockets. My gut feeling is that this could lead to a procedure to find the minimum lift-off speed of a rocket.

What else do we need to know about the Natural Frequency?

A high Natural Frequency means that the rocket can turn very quickly. This can present some problems with RockSim trying to calculate the flight path of the rocket. The way RockSim works is to break the flight up into very small time increments. At each time slice, the software calculates the rockets position, velocity and orientation. If the rocket can change directions so quickly, the rocket could be pointed in a different direction than what RockSim thinks it should be pointed in. So the next time increment is going to start with bad information.

The solution to this dilemma is to make sure the time increment is small enough that the software can catch the direction change before it creates big errors. In practical terms, this means to make sure the time-increment step size, called the "simulation resolution" (found on the simulation controls tab when prepping a rocket for flight) is at least twice as great as the Natural Frequency of the rocket. See Figure 6.

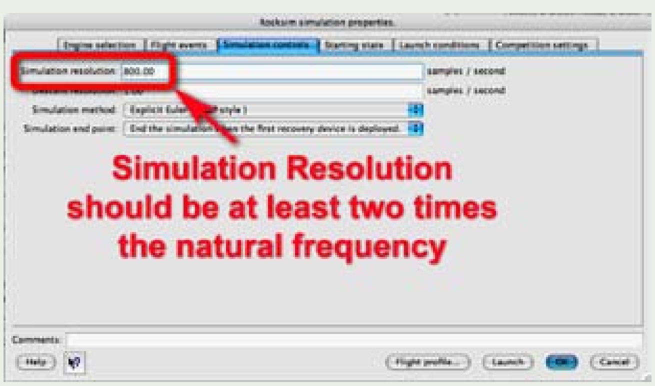

Figure 6

Figure 6: The simulation resolution needs to be at least twice as big as the Natural Frequency for RockSim to be able to catch the direction changes of the rocket because of oscillations.

By default, the sample resolution in RockSim is 800 samples/second. This is fine for most rockets. As you can see in Figure 3 above, the maximum Natural Frequency of the Apogee Avion kit with a C10 motor is less than 180 radians/second. So the simulation resolution in this case is 4.4 times the largest Natural Frequency. The simulation resolution can actually be turned down a little bit, and this will help speed up the time it takes RockSim to perform all the calculations for a single launch simulation.

I rarely recommend that anyone needs to turn up the simulation resolution past 1000. In fact, I can only recall one single instance where 1000 samples/second was too low. In that case, the rocket was basically a huge upscale of the Big Daddy rocket kit. Since it was a big rocket, the Corrective Moment Coefficient was also a huge value. However, since it was relatively short, the Longitudinal Moment of Inertia was fairly small. Therefore, the Natural Frequency of that particular rocket was 1400 rad/sec (which is a very high frequency).

When a simulation was run, the rocket would always go unstable, even though it had an adequate static stability margin. RockSim wasn't catching the correct orientation of the rocket, since it should have been swinging back in its oscillations. RockSim thought it should be spinning round-and-round.

The simple fix of increasing the simulation resolution to 3000 was enough to get the software to adequately calculate the trajectory of that particular rocket. But the downside of that was that it took a very long time for RockSim to completely calculate the trajectory of the rocket.

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I don't recommend that anyone turn up the simulation resolution past 1000 samples/second. As I've shown above, going past 800 is usually overkill and it slows down the processing time needed to calculate a rocket's trajectory.

Conclusion

In this article, we talked about the Natural Frequency of the rocket. We learned what it does, how it is calculated, and how we can use it to optimize our rockets for achieving the highest possible altitudes.

In a future Peak-of-Flight e-zine newsletter, I'll discuss the next parameter that will give us some additional insight into the dynamic characteristics of your rocket. And with it, we'll get another design criterion that will help us optimize the design for peak performance.

In the mean time, I highly recommend digging in the pages of the book: Topics in Advanced Model Rocketry. If you don't have a copy, you can order it from the Apogee Components web site at: http://www.ApogeeRockets.com/topics_advanced_model_rocketry.asp

About The Author:

Tim Van Milligan (a.k.a. "Mr. Rocket") is a real rocket scientist who likes helping out other rocketeers. Before he started writing articles and books about rocketry, he worked on the Delta II rocket that launched satellites into orbit. He has a B.S. in Aeronautical Engineering from Embry-Riddle Aeronautical University in Daytona Beach, Florida, and has worked toward a M.S. in Space Technology from the Florida Institute of Technology in Melbourne, Florida. Currently, he is the owner of Apogee Components (http://www.apogeerockets.com) and the curator of the rocketry education web site: http://www.apogeerockets.com/education/. He is also the author of the books: "Model Rocket Design and Construction," "69 Simple Science Fair Projects with Model Rockets: Aeronautics" and publisher of a FREE e-zine newsletter about model rockets. You can subscribe to the e-zine at the Apogee Components web site or by sending an e-mail to: ezine@apogeerockets.com with "SUBSCRIBE" as the subject line of the message.

T.A.R.C. Tip

How do you know if your payload bay is big enough to carry your egg?

You can get a good estimate of how much room you'll need in your payload bay when you're creating your design in the Rocksim software. Lynn Bryant brings us this great RockSim tip:

"Our usual methodology for simulating eggs is to use an Inside Tube, dial in the OD and length we need, dummy up some material and ID combination that will give us the mass we're looking for, and call it an egg. This system gives us a visual with a length and diameter we can use to check padding thickness instead of just an [M] symbol in the middle of a tube."

DEFINING MOMENTS

"Parachute Riser"

A parachute riser is a string or rope that attaches between the parachute's suspension lines and the payload of the parachute.

If a payload is too close to the canopy of the parachute, the payload can create a wake in the air that can interfere with the efficiency of the parachute. As much as 25% of the parachute's drag can be lost by the forebody wake effect. Keeping a sufficient distance between the payload and the parachute canopy minimizes this loss of efficiency.

A good rule of thumb is that the distance be equal to the diameter of the parachute. Longer is better yet. But if the suspension lines on the parachute get too long, it is easier for them to get tangled during parachute packing.

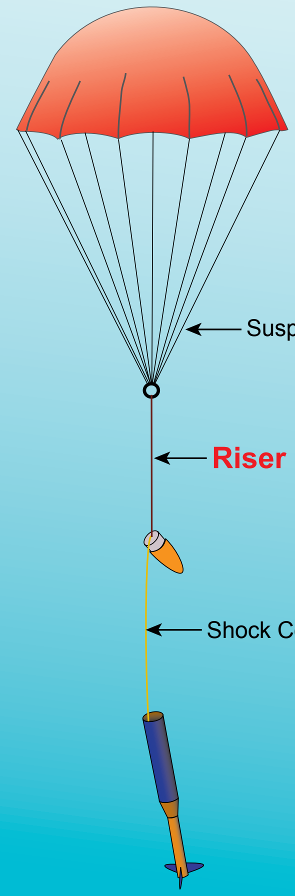
A riser line between the parachute and the suspension lines can help increase the distance without making all the suspension lines longer.

For more information on using riser lines, see Peak-of-Flight Newsletter #187. http://www.ApogeeRockets.com/education/downloads/Newsletter187.pdf

Related Information:

Purchase Parachutes – http://www.ApogeeRockets.com/parachutes.asp

Parachute Design – See Model Rocket Design and Construction. http://www.ApogeeRockets.com/design_book.asp



Apogee News

By Tim Van Milligan and Michelle Mason

This past weekend we held our first rocketry workshop in our new facility. While the attendee list was small (about half-dozen) everyone that came said that they learned something new.

The format of this workshop was quite a bit different than what we've done previously. In prior workshops, we helped rocketeers build the rocket models they brought in. This time, we focused more on teaching the correct construction methods that a new modeler would need when building a simple model.

In order to complete the construction of the rocket in less than an hour, I pre-built 16 models in the last two weeks. Each one showed a particular building skill that a modeler must learn to be able to make a super-quality rocket. We also video-taped this presentation so that we can share it with other modelers. Our goal is to turn this into a DVD that we can get into the hands of new modelers. Look for it sometime in the near future.

Last week, we also released our new educational information packet. It is designed to give teachers the informational resources they need to be able to teach rocketry. It turned out to be huge, over 100 pages long and 15 MB in size. We've already have received some outstanding praise for the depth and the quality of in the information contained in it. The product can be downloaded free, but I would ask that you make a pledge to buy your rocketry products from Apogee Components. This is the only compensation that I ask for. This new product is called the "Rocketry Reservoir," and it can be downloaded at: http://www.apogeerockets.com/Education_Pack.asp

Other than the workshop, it has been fairly quiet here at Apogee. This past Wednesday we had a pizza party as congratulations for the Educational Pack's launch, to kick off the craziness of the holiday season, and of course, to celebrate Halloween.

We are also working on several new projects, so keep up to date by checking the newsletter for great deals, new products, and changes that will be coming up in the next few months!