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Mathematics in Amateur Rocketry

By Joel L. Schiff

{Editor's Note: We often get requests from teachers looking for more information on using rockets in their classrooms. Here is an article that might give you some ideas on how to integrate your math class with rocketry!}

Since you find mathematics in virtually aspect of life, you would naturally expect to find it in the field of 'rocket science'. However, since most amateurs are not launching their rockets into Earth's orbit (not yet anyway), most of the mathematics that the amateur rocketeer has to contend with requires no more than that we all learned in high school (and perhaps have forgotten). So here are some of the basic mathematical tools that will enrich the amateur rocketeering experience by explaining some of its mathematical mysteries.

Calculating Altitude

The most basic of mathematical tools can be used to measure the altitude of a rocket that does not go out of sight. In these days of inexpensive altimeters, using elementary trigonometry to find a rocket's altitude is reserved mainly for low-power rockets. If the rocket goes absolutely vertically, then you can simply measure the distance from the observer to the rocket launch site, and with some measuring device that can measure angles, say a large protractor with a movable arm, measure the angle θ from the horizontal plane to the point at apogee. Then the maximum altitude h is simply given by the equation:

$$h = b \tan \theta$$

where b is the horizontal distance to the rocket's launch position on the ground.

But very often the rocket does not go straight up, and in this case we will need two observers. Here we have the following scenario with observers stationed at positions A and B , separated by a straight-line distance L as shown in Figure 1.

Note that now we do not know the distance b , and it is this quantity, along with the angle θ , which need to be determined. To this end, the distance L between the two observers must be measured beforehand, and then at apogee a verbal signal must be given so that both observers perform their measurements at the same time.

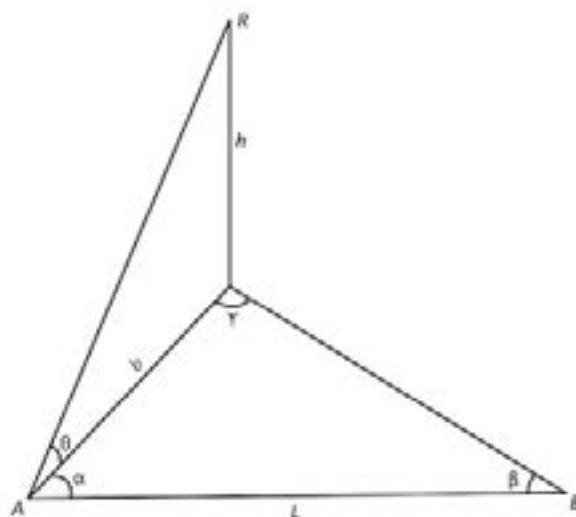


Figure 1: Basic layout of a two-station tracking system.

To keep things as simple as possible only one observer needs to make two measurements. In this case say, observer A measures the altitude angle θ (as in the simpler preceding case) and also the angle α in the horizontal plane from the line L to the rocket's position. Observer B only need measure the angle β in the horizontal plane to the rocket's position.

As previously we can write

$$h = b \tan \theta$$

and the task at hand now is to again determine the value of b . To this end we note that the angle γ is just $180 - (\alpha + \beta)$. The Law of Sines says that

$$\frac{b}{\sin \beta} = \frac{L}{\sin \gamma}$$

and as $\gamma = 180 - (\alpha + \beta)$, we can write

$$\sin \gamma = \sin(180 - (\alpha + \beta)) = \sin(\alpha + \beta)$$

Substituting this into the preceding equation gives us the desired expression for b .

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$$b = \frac{L \sin \beta}{\sin \gamma} = \frac{L \sin \beta}{\sin(\alpha + \beta)}$$

Now we can write the final formula to calculate the altitude b from our original relationship:

$$h = b \tan \theta = \frac{L \sin \beta \cdot \tan \theta}{\sin(\alpha + \beta)}.$$

Total Impulse

The total impulse of a rocket motor is the total thrust (force) of the motor produced for the duration of its total burn time. The force produced by the rocket motor is a highly variable quantity as can be seen by the typical graphs of the thrust curves of various motors taken from the Aerotech catalog (Figure 2).

As the gas from the rocket motor is accelerated out the back of the engine, by Newton's Second Law an equal force is produced in the opposite direction propelling the rocket upward.

The basic formulation for the total impulse is given by the mathematical expression

$$I = \int_0^T F(t) dt$$

which can be understood in the usual calculus sense as

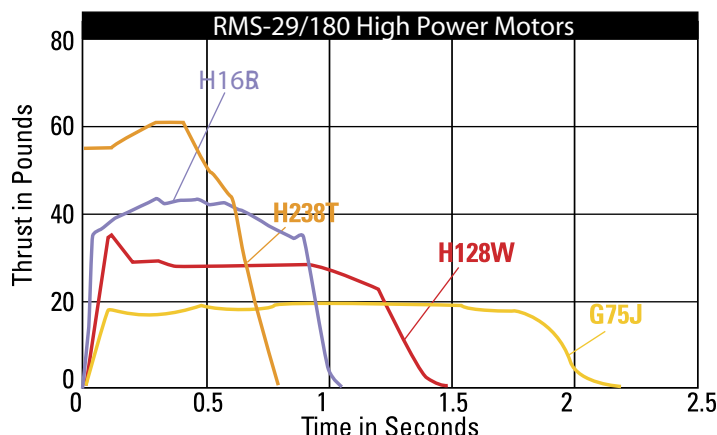
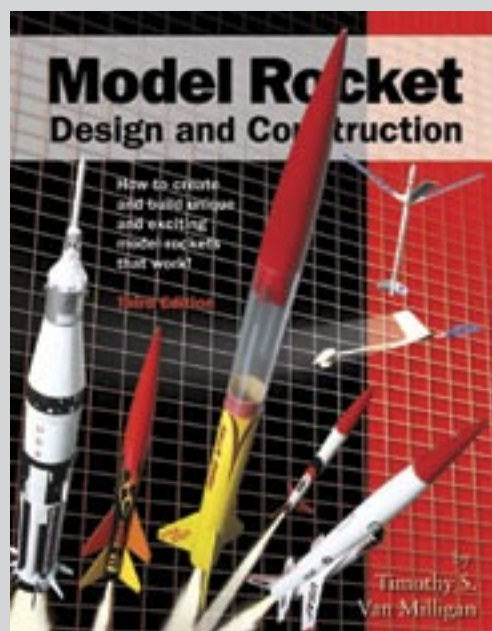


Figure 2: A typical thrust curve shows how variable the thrust force is during the burn of the motor.

representing the total area under the thrust curve defined by the thrust function $F(t)$, over the burn time interval $[0, T]$, the value being given in units of Newton-seconds, or N-sec. Actually, in most instances for rocket motors, the function F is not something that can be explicitly represented, and so one can measure the area under the curve directly by cutting out simple shapes that can be measured and used as an approximation. Fortunately for us, manufacturers provide this information.

We should point out that the 'burn time' for the purposes of 'average thrust' is not the total burn time from one end of the thrust curve to the other. It is a truncated version of the total elapsed time where the initial 5% and final

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5% of the maximum thrust have been dropped off the total impulse graph. And dividing this truncated impulse by the truncated burn time gives the 'average thrust', a rounded off version of which is printed on the side of the rocket motor. This is the reason that if you simply multiply the 'average thrust' on the rocket motor, say the 300 from an Aerotech I300, times its total burn time which in this case is 1.6 seconds, you will not get the 'average thrust' of 440 N-sec as stated by Aerotech. Now you know why.

A related notion to total impulse is that of specific impulse, I_{sp} , which is the total impulse divided by the mass of propellant (the units come out in seconds). It gives a measure of how effective the propellant actually is at producing thrust. For example, the specific impulse of black powder is 80 – 90 sec, for sugar/potassium nitrate it is about 130 sec, and for ammonium perchlorate composite propellant (APCP) it's 190 – 210 sec. And this is one of the reasons why APCP is used in most commercial amateur rocket motors as well as in the Space Shuttle Boosters. It gives a lot of bang for your buck.

Thrust-to-Weight Matters

This is a very important issue since many a rocket has become unstable simply because there has not been enough thrust to keep it on course. The usual recommended T:W (thrust-to-weight) ratio is 5:1 which allows for a healthy margin of error in some of the details.

The first thing we note is that weight is usually mea-

sured in ounces/pounds or grams/kilograms and that thrust is measured in Newtons. A Newton is the force required to accelerate a mass of 1 kilogram (2.2 lbs) at a rate of 1 meter per second per second. Since this is not very intuitive, it turns out that a one-kilogram mass pushes down on the surface of the Earth with a force of roughly 10 (actually 9.8) Newtons. And yes, it was named after Sir Isaac Newton.

Thus if we take our rocket motor's average thrust in Newtons and divide by 9.8 we will obtain kilograms, which is a measurement of weight, as we desired. If we instead want pounds of thrust, since we know that 1 kg = 2.2 lbs, we find that a 1 lb mass pushes down on the Earth's surface with a force of $9.8/2.2 = 4.45$ Newtons.

In order to calculate the thrust-to-weight ratio, we take the *average* thrust of the rocket motor which is given in Newtons, and divide it by 4.45. This will give us the lift capacity of the motor in pounds. As we want this capacity to be 5 times the weight of the rocket, we can divide this lift capacity by 5 and check to see if it exceeds the weight of the rocket fully loaded (including its motor).

Let's see how this works in practice. Suppose we wish to use an I154 motor in our rocket that in turn gives it a loaded weight of 88 oz. We take the average thrust to be 154 Newtons and dividing this by 4.45 converts to 34.6 lbs. Dividing this in turn by 5 yields 6.9 lbs which represents the 5:1 liftoff weight capacity. As our loaded weight of 88 oz equates to 5.5 lbs, we are comfortably within the 5:1 thrust-to weight ratio.

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The 5:1 thrust-to-weight algorithm can be summarized thusly:

$$(\text{average thrust}/4.45)/5 > \text{loaded wt of rocket (lbs)}$$

If this is not the case, either shed some weight or use a more powerful motor.

Static Port Size and Multiple Ports

The calculation of the size of a single static port is based on a bit of rocketry folklore. Namely, a port diameter of 1/4-inch is required for every 100 cubic inches of volume of your avionics bay. This rule of thumb has served amateur rocketeers very well and is the basis for our subsequent discussion.

In order to compute the av bay volume, we need the following formula for the volume of a cylinder that most of us learned at school as some stage, namely

$$V_{av} = \pi r^2 h$$

where V is the volume of the bay, r is its radius, and h is its length. Let's rewrite this in terms of the diameter D of the bay and call its length more appropriately L :

$$V_{av} = \pi (D/2)^2 L$$

Most likely, the volume of your avionics bay will not turn out to be 100 cubic inches and so we must interpolate from our rule of thumb. The basic notion here is that if we say double or halve the volume, then the area of the port hole

should be doubled or halved. This means that the area of the static port hole is proportional to the volume of the av bay, that is,

$$A_{hole} \sim V_{av}$$

i.e.

$$A_{hole} = k V_{av}$$

where k is some constant of proportionality yet to be determined.

Denoting by d the diameter of the port hole, this equation becomes

$$\pi (d/2)^2 = k \pi (D/2)^2 L$$

and after cancellation we have

$$d^2 = k \cdot D^2 L$$

And so the hole diameter is given by

$$d = \sqrt{k} \cdot D \sqrt{L}$$

To utilize this equation we must find the constant k , which can be done since we know that a diameter of 1/4 inch suits a volume of 100 cu in. So the equation

$$A_{hole} = k V_{av}$$

gives

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$$\pi \cdot \left(\frac{1/4}{2}\right)^2 = k \cdot 100$$

and solving this for the constant k gives,
 $k = 0.00049$ and thus $\sqrt{k} = 0.022$.

Finally, our formula for reads

$$d = 0.022D\sqrt{L}$$

and this is the formula for the diameter of the single static port in terms of the diameter D of the avionics bay and its length L . Interestingly, Entacore recommend the constant be 0.01 in the preceding equation for their AIM USB altimeter (www.ApogeeRockets.com/entacore_aim_usb.asp).

Now let us consider the case of more than one static port, since there is safety in numbers in this instance.

Denote by A_N the size of each of N ports. Then what we want is for the total area of the N ports to add up to the same area as the single port, that is

$$N \cdot A_N = A_1$$

Using the familiar $N = \pi r^2$, and denoting d_N as the diameter of each of the N ports, with $d_1 (=d)$ the diameter of a single port as given in the discussion above, the preceding equation reads:

$$N \cdot \pi \cdot \left(\frac{d_N}{2}\right)^2 = \pi \cdot \left(\frac{d_1}{2}\right)^2$$

Canceling the π and 4 on each side yields

$$N \cdot (d_N)^2 = (d_1)^2$$

and taking square roots of each side and writing in terms of d_N ,

$$d_N = (1/\sqrt{N})d_1$$

If we now substitute in the value of $d_1 (=d)$ from our preceding discussion of a single port, we obtain the general formula for the diameter d_N of each of any number of ports,

$$d_N = 0.022D\sqrt{\frac{L}{N}}$$

where D is the diameter of the av bay, L is its length, and N the number of static ports desired.

Actually, well-known rocketeer Vern Knowles recommends 3 static ports (or 4) rather than 2, so in the case of 3 ports, the former equation becomes (for $N = 3$):

$$d_3 = 0.577 \cdot d_1$$

and for $N = 4$, $d_4 = 0.5d_1$. This makes good sense as taking half the diameter is also taking half the radius, and circles with half the radius as the original have 1/4 the area of the original. So having 4 such circles gives the same total area as the original single circle.

Black Powder Requirements

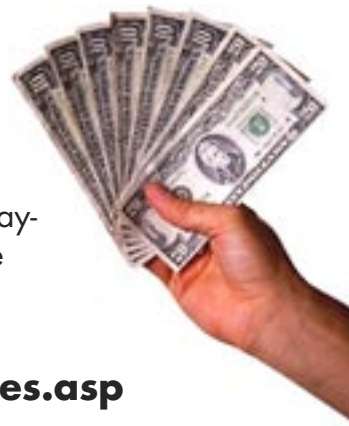
Another important mathematical consideration of the amateur rocketeer is the question of how much black powder is required to blow off a closed compartment section of the rocket. This is usually done using 4F grade (FFFF g) black powder which has the smallest size grains and are therefore the easiest to ignite. Ignition produces a significant volume of gas which in turn over-pressurizes a closed compartment and if the force is sufficient, something has to give, like a nose cone or payload section.

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So we consider the Ideal Gas Law equation to determine our black powder requirements. It governs the state of an 'ideal' gas and is given by the relationship

$$pV = nRT$$

where p represents the gas pressure, $V = \pi(D/2)^2L$ the volume of the pressurized compartment having diameter D and length L , n is the mass of the substance, R the gas constant, and T the gas temperature.

As a general guideline, we will take the pressure to be a nominal $p=15psi$ (pounds per square inch). For black powder the combustion temperature is $T=3300^\circ F$ and the constant $R=266$ in the present scenario. Solving for n in the above and putting in the appropriate values gives

$$n = \frac{pV}{RT} = \frac{15 \cdot \pi(D/2)^2 L}{266 \cdot 3300}$$

which gives the value of n in lbs. In amateur rocketry we usually deal only with small amounts of black powder measured in grams, so we need to multiply the above value of n by 454 g/lb. When we carry out the whole calculation we arrive at a value for n in terms of the dimensions of the pressurized compartment,

$$n = 0.006D^2L$$

This figure is only a guide to how much black powder to use (although a good one) and a true ground test is

mandatory because there are many extenuating factors. In fact, for larger rockets, i.e. those having a diameter greater than about 5.5 inches, the overall pressure on a bulkhead becomes far too great and needs to be kept within bounds (say, less than 350 lbs of total force) by suitably modifying the above equation.

This pretty much covers the basic mathematical background that you need to know in order to leave Earth with confidence and understanding. The rest is up to you.

About the Author

Joel Schiff grew up in Los Angeles and has a PhD in Mathematics from UCLA. He also taught there but spent most of his career at the University of Auckland, Auckland, New Zealand. He is the author of three books, co-discover (with his wife) of asteroid 12926 Brianmason, and former editor/publisher of the journal *Meteorite*. He is now obsessed with rockets.



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