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NEWSLETTER

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Finding the Formula for a Parabolic Transition Section

By Tim Van Milligan

You know teenagers... They complain all the time about schoolwork. The common question they utter when they complain is: *"when am I ever going to use this information in real life?"* They assume that if you can't come up with a good example, then it would be a tacit approval for them to skip the assignment and go play games on their smart phones. My daughter is no different, even though she has been flying rockets since she was about 5 years old and she should know better. She's taking geometry now, and wonders when she'll ever have to plot out weird shapes, like a parabola.

I came across a situation and I thought I'd document it in this article, so that I can prove to her that what she's learning in school has a practical application. The situation I'll be discussing is making a real parabolic transition section and nose cone.

Why Do I Need A Parabolic Transition Section?

This summer, she is traveling to the World Space Modeling Championships in Europe, and one of the events is A-engine altitude. The event rules are different from the NAR's rules for altitude, in that there is a minimum size of the rocket. They found out that the rockets fly so high that they are nearly impossible to see. To make sure they can be seen, the rules dictate a significantly large size rocket. The minimum length of the rocket is 500mm, and half of that length must be at least 40mm in diameter. To put that into perspective, imagine a rocket about the size of an Estes Big Bertha. Now remember, it has to fly on an "A" engine which has 2.5 N-s of power. As you can imagine, models the size of a Big Bertha don't fly very high on "A-size" motors.

Therefore, you have to go to some extremes to squeeze more altitude out of the model. The first method is to reduce the weight of the rocket. Instead of using thick-walled body tubes, you make them as thin as possible. One of the first things she did last summer, when trying out for Team USA, was to make the tubes out of a single sheet of copy-paper. When you do that, the altitude of the rocket goes way up.

Unfortunately, paper has one very big drawback: it absorbs water. When flying at a humid launch site, our thin-walled rockets quickly became limp as the paper absorbed water from the air. They are almost impossible to pick up without deforming them. And as soon as they landed in dew-wet grass, they were ruined.

The solution to this problem is to make a fiberglass rocket. While fiberglass has a higher density than paper, it doesn't absorb water. That means you can fly them multiple times on humid days.

The other advantage of fiberglass is that you can mold the cloth into curved shapes. This is practically impossible to achieve with paper. With paper, you can't make a curved nose cone, like an ogive or parabolic shape. The closest

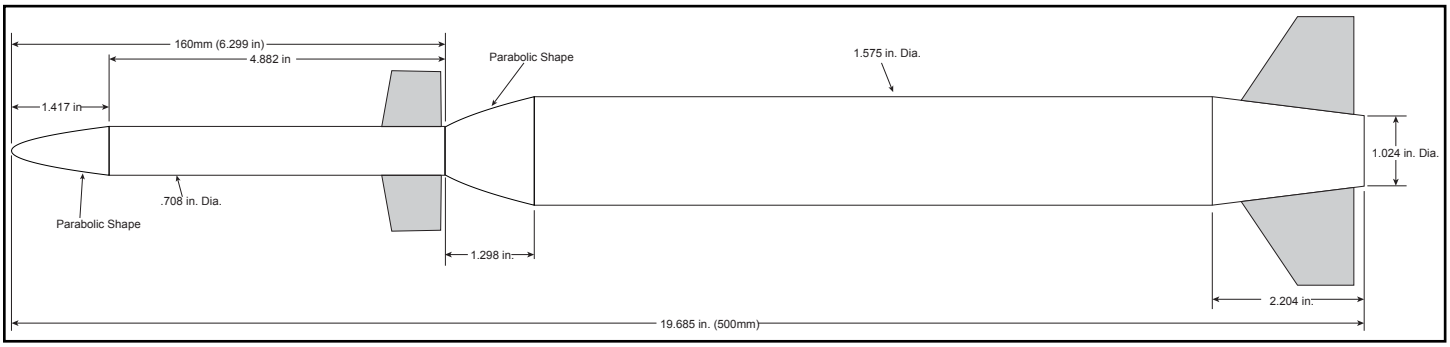

Figure 1: Basic dimensions of the two stage altitude rocket for international competition.

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you can make is a conical shape.

Make The Rocket More Aerodynamic

And making curved shapes is the next trick to use to squeeze more altitude out of a rocket. It lowers the drag of the model. When the drag goes down, the altitude of the rocket goes up proportionately.

This was proved to my daughter last summer, by measuring the drag force on various shaped nose cones at the Air Force Academy (see *Peak-of-Flight Newsletter #346* at: www.ApogeeRockets.com/Education/Downloads/Newsletter346.pdf). The cone shapes were higher drag compared to the rounded shapes, like the parabolic shape and the elliptical shape.

Now, just like in NAR competition, for international competition you are allowed to stage the rocket. And to be competitive, you almost have to. The advantage of staging is that you can drop off a lot of mass to make the model travel higher. And the rules allow that you can drop the fat portion (that 40mm diameter portion) and just have a very small sustainer that travels really high.

What this means is that there will be some sort of transition between the skinny part of the rocket, and the fat section. And guess which shape the transition should be to minimize drag? You guessed it, the curved parabolic shape.

Now that you know the shapes to use, it is only a matter of determining what the lengths of the components should be to minimize the size and the weight of the upper stage. There is a very good article showing the trade-offs in sizing the various components of this type of rocket at: https://sites.google.com/site/xfaispacemodeling/wsmc-events/s1---altitude

Figure 1 on page 2, shows the dimensions of the rocket that we settled on for this contest event. Actually, I copied the dimensions from a Russian design. They have a long history of success in this event, so I felt it best to piggyback on that design.

With the dimensions of the rocket sorted out, the next challenge is to figure out how to build the rocket out of fiberglass. The one disadvantage of fiberglass rockets is that they require a lot of tooling to form the cloth. To be honest, when all is said and done, I'll probably have spent over $500 on mandrels and other equipment to make the rockets. Fortunately, unless the rules change, this is a one-time expense and she can make as many airframes as she wants with only the extra cost being a few bucks for the fiberglass cloth and the epoxy resin.

This gets us back to my daughter's complaint that geometry has no real-life applications. I need to provide the equation of the parabola for both the nose cone and the transitions. Now this isn't actually necessary with CAD programs or RockSim (www.ApogeeRockets.com/RockSim/

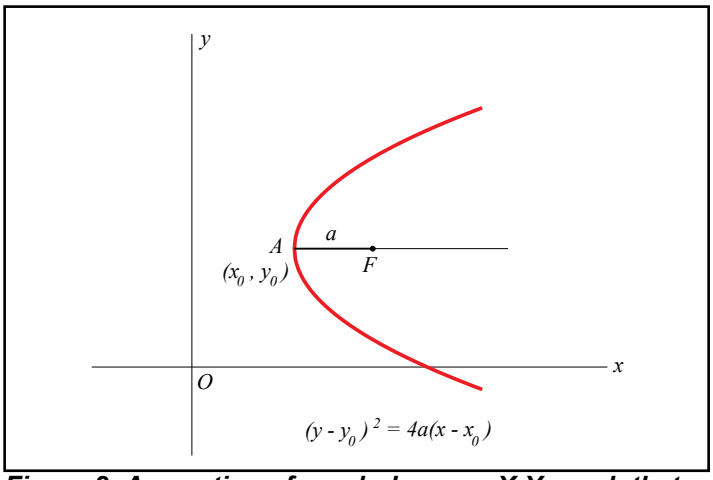

Figure 2: A equation of parabola on an X-Y graph that opens to the right.

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RockSim_Information), as they will often figure out the equations on their own. But I wanted to show my daughter how to the parabolic shape is calculated and how the automated lathe would cut the part.

I'll start with getting the formula for the nose cone, and then we'll get the one for the transition secton.

Back To The Textbook

I dug out my old math book, and found the formula for a parabola. This is shown in Figure 2.

The formula for the points along the parabola (that opens to the right on a X-Y graph) is given by the equation:

$$(y - y_0)^2 = 4a(x - x_0) \qquad \text{eq. 1}$$

The first thing we want to do is move the tip of the parabola to the Origin of the graph (point O in Figure 2). When you do that, both x_0 and y_0 are zero. That simplifies the equation to:

$$(y - \cancel{y_0}^{\,0})^2 = 4a(x - \cancel{x_0}^{\,0}) \qquad \text{eq. 2}$$

$$y^2 = 4ax \qquad \text{eq. 3}$$

The one unknown is the "a", which is the distance from the tip point to the focus of the parabola. So we'll have to solve for that distance. This gives us the equation:

$$a = \frac{y^2}{4x} \qquad \text{eq. 3}$$

Next, we'll need a point on the parabola so that we can solve for a. Fortunately, we do have one point, it is where the nose meets up with the body tube of the rocket. As

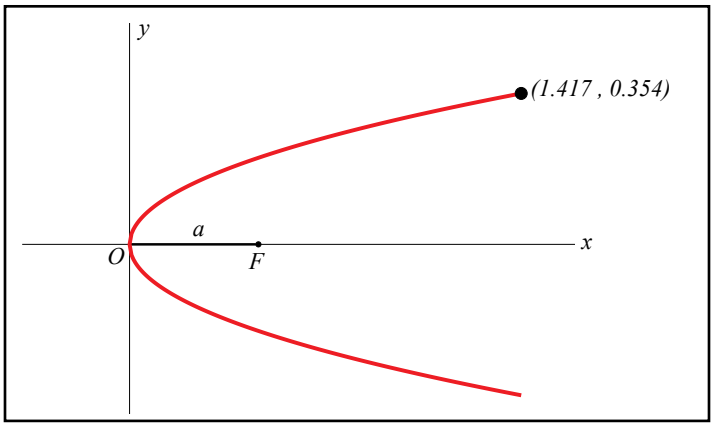

Figure 3: The top-right corner of the parabola is where it intersects the body tube. The y distance is the radius of the tube.

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shown in Figure 1 on page 2, the nose is is 1.417 inches long, so this is the x distance. The y distance is the radius of the tube where it touches the nose cone. The diameter of the tube is .708 inches, so the radius must be 0.354 inches (see Figure 3). Plugging these numbers into the equation allows us to solve for the focus distance, a.

$$a = \frac{0.354^2}{4\,(1.417)} \qquad \text{eq. 4}$$

$a = 0.022109$ inches

This can then be plugged back into equation 2, which allows us to find the formula for any point along the curve of the parabola.

$$y^2 = 4(0.022109)x \qquad \text{eq. 5}$$

$$y = \sqrt{4\,(0.022109)x} \qquad \text{eq. 6}$$

$$y = \sqrt{0.08437x} \qquad \text{eq. 7}$$

This is the formula for any point along the nose cone, as measured from the tip. Remember that y is a radius, so if you or your machinist wants a diameter, the value must be doubled.

From this equation, I used a spreadsheet program to create X,Y coordinates for the shape of the nose, and sent them off to the machinist to make my metal mandrel that is used to make the nose cone. The more points you have, the smoother the curve. I used 50 points on the nose cone, where the distance (x) apart was .028 inches. That is pretty close together, and didn't take much effort to smooth out the surface. Figure 4 shows the complete mandrel for the nose section.

Figure 4: The aluminum mandrel made using the equation of the parabola found in the text.

Incidentally, if you are wondering how to turn the mandrel into a fiberglass part, see Apogee Technical Publication #12 which can be ordered at: www.ApogeeRockets.com/Rocket_Books_Videos/Pamphlets_Reports/Tech_Pub_12.

Finding the Formula For A Parabolic Transition Section

Finding the formula for the parabolic transition section is a little harder than the nose cone. The reason is that we

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don't know where the tip of the parabola is, and therefore we don't know what the focus distance (a) is.

But we do know two points on the curve of the parabola from the diameter of the the front end and the rear end of the transition. We'll call the front, point 1, and the rear point 2. Figure 5 shows the points on an X,Y graph.

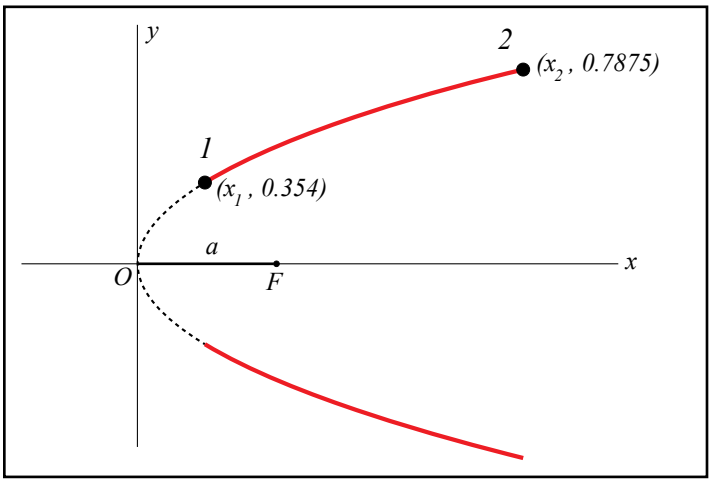

Figure 5: The known information about the coordinates of the parabolic curve of the transition section.

As before, we're working with the radius of the nose cone, so these will be the y coordinates on the graphs.

What we don't know, are a, x_1, and x_2.

We can use the equation 3 for points 1 and 2, but we still have three unkowns.

But there is one other piece of information that we have available. That is the distance between the front of the transition and its rear. That distance, as shown in Figure 1, is 1.298 inches.

We can therefore say that:

$$x_2 = x_1 + 1.298 \qquad \text{eq. 8}$$

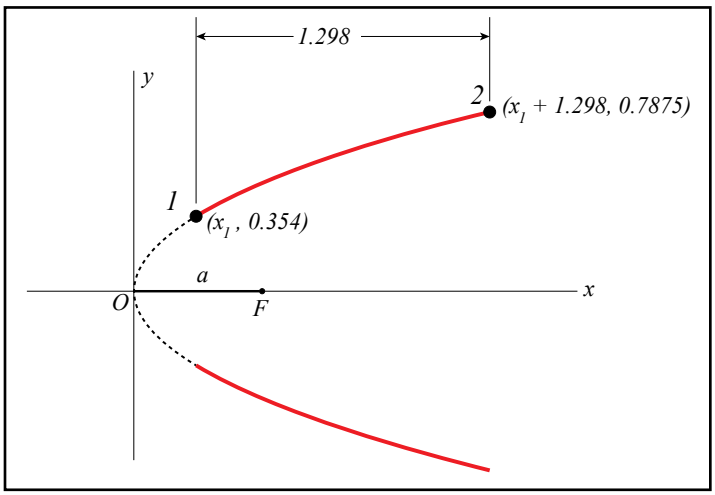

Figure 6: The distance between the points can help simplify the complex equation.

This is shown in Figure 6.

We'll start with the Point 1, and plug its coordinates into equation 3:

$$y_1^2 = 4ax_1 \qquad \text{eq. 9}$$

Substituting the radius at the front of the transition, gives us:

$$0.354^2 = 4ax_1 \qquad \text{eq. 10}$$

Squaring the value of 0.354 yields:

$$0.1253 = 4ax_1 \qquad \text{eq. 11}$$

Dividing both sides by 4 gives:

$$0.0313 = ax_1 \qquad \text{eq. 12}$$

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Finally, we can rearrange to find out what a will be:

$$a = \frac{0.0313}{x_1} \qquad \text{eq. 13}$$

Now we'll solve for Point 2 in Figure 6, by plugging its X.

$$y_2^2 = 4ax_2 \qquad \text{eq. 14}$$

From Equation 8, we'll substitute for X_2:

$$y_2^2 = 4a\,(x_1 + 1.298) \qquad \text{eq. 15}$$

We'll also plug Equation 13 and substitute for a:

$$y_2^2 = 4\left(\frac{0.0313}{x_1}\right)(x_1 + 1.298) \qquad \text{eq. 16}$$

Now we'll start reducing. First, multiply the first term by 4:

$$y_2^2 = \left(\frac{0.1253}{x_1}\right)(x_1 + 1.298) \qquad \text{eq. 17}$$

We'll use the distributive property, and multiply through the terms on the right side:

$$y_2^2 = (0.1253 + 0.1627 / x_1) \qquad \text{eq. 18}$$

We know the value of y_2 from the graph in Figure 6, so we'll plug that into the equation now:

$$0.7875^2 = (0.1253 + 0.1627 / x_1) \qquad \text{eq. 19}$$

Squaring the first term yields:

$$0.6202 = 0.1253 + 0.1627 / x_1 \qquad \text{eq. 20}$$

Subtracting 0.1253 from both sides gives:

$$0.4948 = 0.1627 / x_1 \qquad \text{eq. 21}$$

Solving for x_1 give us this fraction:

$$x_1 = \frac{0.1627}{0.4948} \qquad \text{eq. 22}$$

Finally, solving the division gives the final answer:

$$x_1 = 0.3287 \qquad \text{eq. 23}$$

Determining y2 can be found by plugging this value into equation 8:

$$x_2 = x_1 + 1.298 = 0.3287 + 1.298 = 1.6267 \text{ inches} \qquad \text{eq. 24}$$

To get the final formula for the transition parabola, we only need to find the value of a, the focus length. That is found by plugging x_1 into equation 13:

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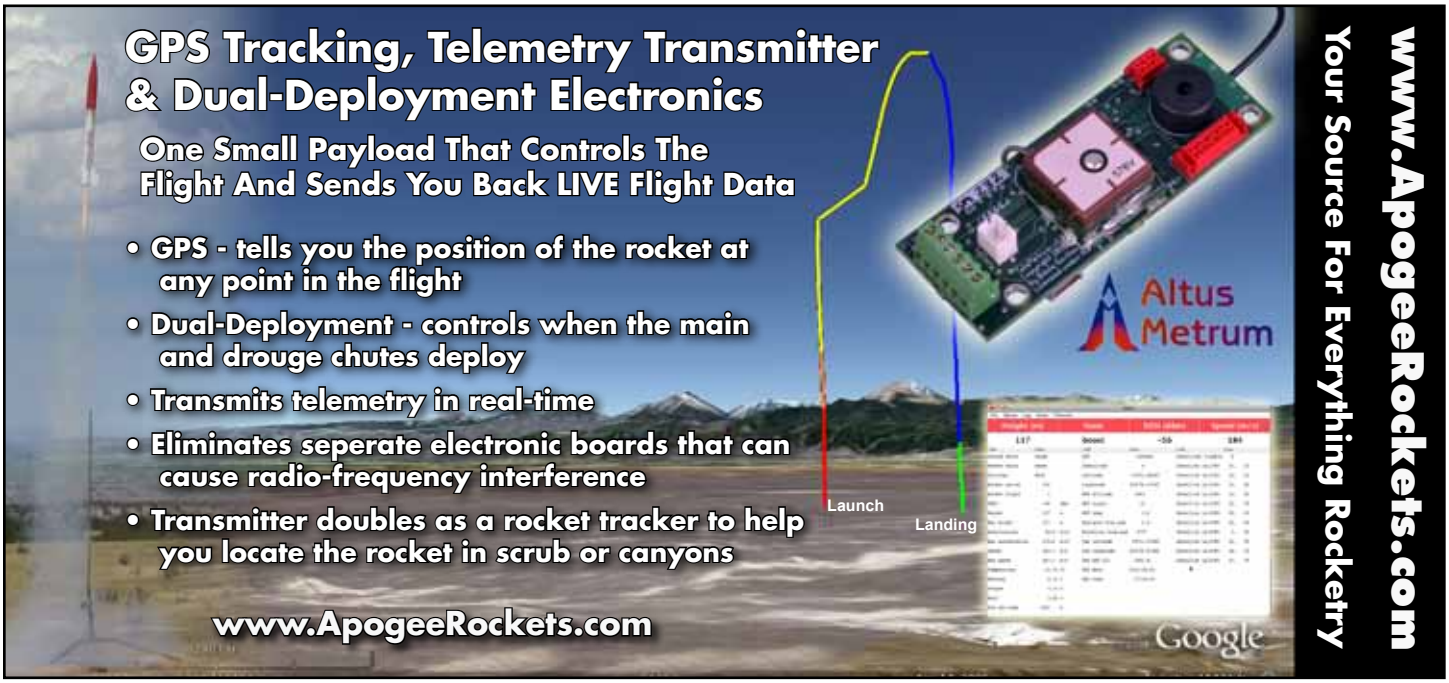

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$$a = \frac{0.0313}{0.3287} = 0.0952 \qquad \text{eq. 25}$$

Now the equation of the parabola is plugging the numbers into equation 9:

$$y^2 = 4(0.0952)x \qquad \text{eq. 26}$$

$$y^2 = 0.38089\,x \qquad \text{eq. 27}$$

$$y = \sqrt{0.38089x} \qquad \text{eq. 28}$$

At this point, we have solved the problem and we're ready to send off the information to the machinist to cut metal. Again, remember that the y value is a radius, measured from the centerline of the part. If you want the diameter at any point along the curve, you'll have to double the value.

Figure 7: The completed fiberglass nose cone

I wish I could show you the picture of the completed mandrel. But unfortunately, I don't have my transition mandrel yet. But it should be coming back from the machinist soon.

Conclusion:

The point of this exercise was to show students that the information they are learning in school has a practical use, and therefore they shouldn't skimp on their studies.

About The Author:

Tim Van Milligan (a.k.a. "Mr. Rocket") is a real rocket scientist who likes helping out other rocketeers. Before he started writing articles and books about rocketry, he worked on the Delta II rocket that launched satellites into orbit. He has a B.S. in Aeronautical Engineering from Embry-Riddle Aeronautical University in Daytona Beach, Florida, and has worked toward a M.S. in Space Technology from the Florida Institute of Technology in Melbourne, Florida. Currently, he is the owner of Apogee Components (http://www.apogeerockets.com) and the curator of the rocketry education web site: http://www.apogeerockets.com/education/. He is also the author of the books: *"Model Rocket Design and Construction," "69 Simple Science Fair Projects with Model Rockets: Aeronautics"* and publisher of a FREE e-zine newsletter about model rockets.