IN THIS ISSUE

Phenomena of Rocket Exhaust Plumes
How many times have you watched a rocket launch on the TV or at your model rocket club and paid just a casual interest in the plume it created? There are a great deal of very interesting things happening in that plume of fire and smoke. In this article we’ll try to break down what you should be watching for at the next televised Space X launch or when you launch your own solid propellant rocket.

But before we explain what those bright white diamond shaped images in supersonic exhaust plumes are or why your solid propellant exhaust is red, blue or white, we should explain some basic physics around rocket exhaust plumes. Isaac Newton’s 2nd and 3rd Laws are the basis for why rockets work at all.

His Second Law we all know is based on the equation Force = Mass X Acceleration. But this is an abbreviated version of what he really wrote in his celebrated treatise Principe de Mathematica. He really said that an object’s momentum is directly proportional to any impulse that acts upon it. In our case that impulse is the burning of propellant and the thrust that results. His Third Law states that for every action on a body there is an equal and opposite reaction. This was met with some skepticism as some believed for a while that rockets required an atmosphere to work and would be useless in the vacuum of space. Of course today this opinion seems ridiculous but it prevailed even as late as the early 20th century.

Before we explain why the exhaust is shaped the way it is and why it has other visual features we must introduce some math. I promise I will keep it minimal but will reference at the end where you can find out more if you’re so inclined.

The thrust from a rocket engine is calculated using the following equation:

\[ \text{Thrust} = F = \frac{m \dot{\text{mass flow rate}} \times \text{velocity of exhaust} + (P_e - P_0) A_e}{A_e} \]

- \( P_e \) = pressure of exhaust
- \( P_0 \) = atmospheric pressure
- \( A_e \) = area of exhaust nozzle

Figure 1: Thrust Formula

The amount of thrust produced by the rocket depends on the mass flow rate through the engine, the exit velocity of the exhaust, and the pressure at the nozzle exit. All of the variables in the equation depend on the design of the combustion chamber and nozzle. All of the exhaust gas is choked at the throat and its speed is Mach 1 (332 m/sec or 717 mi/hr.). The mass flow rate \( m \dot{\text{mass flow rate}} \) (the over dot means the derivative of mass with respect to time) is determined by the throat area. The area ratio from the throat to the exit \( A_e \) sets the exit velocity \( \text{Ve} \) and the exit pressure \( P_e \).
You can see that at a relatively low altitude of 10,000 feet the atmospheric pressure has already dropped by 1/3. Hence, a nozzle designed so that its exit pressure is equal to atmospheric pressure at sea level will be under expanded (exit plume pressure will be more than atmospheric) and the rocket loses efficiency, however it also is lighter at this point. A few more thoughts on compressible fluid dynamics (CFD). The velocity of a rocket nozzle increases with its area. This sounds counterintuitive because you may have thought that the velocity decreases with area increase but this is only true for incompressible fluids. However, whether a fluid is compressible or not, the velocity and pressure are always inversely proportional. That is, when velocity increases the pressure will decrease. The flow will be subsonic in the combustion chamber and it will reach sonic speed when passing through the throat. In the diverging portion of the nozzle it reaches supersonic velocities.

Figure 2: Exhaust plume chart

There are 3 possible pressure conditions for the exhausting plume of gas. The one that delivers the maximum thrust is when the pressure of the exhaust equals the ambient atmospheric pressure. All three conditions are illustrated in Figure 2.

The next time you watch a Space X launch observe the exhaust plume. Most rocket nozzles are designed to be either slightly less than atmospheric pressure (over expanded) at launch or equal in order to maximize thrust when the rocket is loaded with fuel and is the heaviest. The design of the nozzle diameter on the first stage rocket will usually become under expanded as the rocket gains altitude (see the far right drawing, Figure 2) due to the exponential decrease in atmospheric pressure (see Figure 3).
Over expanded nozzles produce shock diamonds which we will discuss next. Remember an over expanded nozzle is when the velocity is greatest and pressure is less than atmospheric pressure. Fighter jets will have over expanded nozzles so that there is no damage to the airframe from the exhaust plume.

As an object moves through the air, the air molecules are deflected around the object. If the speed of the exhaust is much less than the speed of sound of the gas, the density of the gas remains constant and the flow of gas can be described by conserving momentum and energy. As the speed of the exhaust increases towards the speed of sound, the compressibility of the exhaust gas comes into play. The density of the exhaust varies locally as the gas is compressed by the atmospheric pressure. When an exhaust moves faster than the speed of sound, and there is an abrupt decrease in the flow area, and shock waves are generated. The waves that form in the supersonic exhaust have the same constructive and destructive properties that waves in a pool have. A shock wave forms where the constructive crests of the wave meet (see Figure 4).

The billions of individual sound waves pile up, crest on crest, forming a cone that follows behind the supersonic jet or in the rocket’s exhaust plume. This is why you hear a sonic boom after the supersonic aircraft has passed you (see Figure 5).
There are several types of shock waves that can form: bow shocks, attached bow shocks, oblique shocks, and normal shocks.

Bow shocks are detached from the object that is creating them. If bow shocks become attached due to the shape of the object making them, they will create drag, something to be avoided for aircraft and fighter jets. This is one reason supersonic aircraft, whether a F-22 or the retired Concorde, are designed with a pointed rather than blunt nose.

Oblique shock waves form when the supersonic flow is rapidly redirected. These types of waves will speed up the flow. Oblique shock waves form as the supersonic exhaust leaves rockets nozzle.

Normal shock waves form perpendicular to the flow and they will change the flow from supersonic to subsonic.

We will introduce one other phenomenon called Prandtl-Meyer Expansion Fans during the discussion to follow on shock diamonds.

Pardon my digression, but a word about oblique and normal shock waves as they were used for the air intake on the fastest jet ever built, the SR-71 Blackbird (Figure 6).

A relatively unknown design feature of the supersonic SR-71 Blackbird J58 engines are that the inlet spikes or center body located in the nacelle of the jet engine intakes were put there to control the oblique shock waves. Moreover they were put there to take the pressure of the supersonic shockwave off of the engines. The spikes take the oblique shock waves and convert them to a normal shock wave. The air entering the nacelle is converted from low pressure supersonic speeds to high pressure subsonic speed. The spike retracts 1.6 inches for every 0.1 increase in Mach number for speeds above Mach 1.6 to keep the normal shock in the optimum position inside the inlet. The cruising speed for the Blackbirds is Mach 3.2 and the spike has retracted 26 inches at this speed. They are carefully designed so the oblique shocks fall just inside the air inlets, otherwise the drop in pressure would ruin the efficiency of the engines.

I hope you didn’t mind this short detour into the way the SR-71 engines control the shock waves created as a result of flying at 2,200 mph. It was a truly remarkable aircraft that was designed and built before calculators and computers. That’s correct; this spy plane from the 50’s & 60’s was built using slide rules!
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shells. Mach mathematically proved that the shell when near the speed of sound created a disturbance by compressing the air locally and changing its density. The properties that change are static pressure, temperature and density. Of course, besides the diamond shaped feature being named in honor of him the Mach number (ratio of object speed to speed of sound) also bears his name. A side note is that Marilyn vos Savant, who holds the Guinness record for being tested with the highest IQ, is a descendant of Dr. Mach.

Along with the double report the phenomena of bright flashes seen at night from artillery shells spurred scientific research on what caused them and how to eliminate them since they gave away the position of the guns. They wanted to design a gun to eliminate the flash and realized that a crucial parameter was the ratio of the pressure in the exhaust gas to the ambient pressure. They flared the nozzle slightly, lowering the pressure and eliminated the flashes which were actually Mach disks.

The shock diamonds from rocket exhaust form when the supersonic exhaust is slightly over expanded, meaning the static pressure of the gas is less than ambient air pressure. This pressure increase is adiabatic (meaning there is no gain or loss of heat to the surroundings) and the Mach disk or diamond will result in any unburned combustion products to reignite. Many of you who are old enough remember the SR-71 Blackbird at take-off with a dual “necklace” of shock diamonds from its engines. The exit pressure is less than atmospheric pressure (over-expanded), so the pressure of the exhaust is compressed.

Figure 7: Engine test with Mach Diamonds
Finally, on to the discussion of Shock Diamonds.
These diamond shaped features are found in the exhaust plume of not only rockets but also supersonic aircraft.

We will start with a brief history on how they were discovered. The diamond shaped features known as Shock Diamonds or Mach diamonds were named after Ernst Mach, a late 19th century German physicist who studied gas dynamics. Mach became interested in this field when during the Franco-Prussian War artillery experts were groping for an explanation of what caused a double shock report from high speed artillery shells. Mach mathematically proved that the shell when near the speed of sound created a disturbance by compressing the air locally and changing its density. The properties that change are static pressure, temperature and density. Of course, besides the diamond shaped feature being named in honor of him the Mach number (ratio of object speed to speed of sound) also bears his name. A side note is that Marilyn vos Savant, who holds the Guinness record for being tested with the highest IQ, is a descendant of Dr. Mach.

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Figure 8: Mach Diamonds on SR-71

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The flow generates waves which form the basis of supersonic fluid dynamics. Among these waves are some that are oblique and some are normal (90 degrees) to the exhaust flow.

The diagram below comes from aerospace-web.org. In addition to the oblique and normal shock waves a so-called expansion fan develops. As the supersonic exhaust reaches the farthest edge of the nozzle, oblique shock waves are created and the gases are directed toward the center of the plume. The oblique (angle is less than 90 degrees) shock cannot penetrate the stream of gasses in the center and they are reflected back toward the plume’s outermost boundary. As they encounter the boundary the pressures between these supersonic waves and the ambient pressure are quite different setting up another reflection that this time produces an expansion fan of waves known as Prandtl-Meyer phenomenon. The expansion fan also cannot pass through the center stream of particles and is reflected again back toward the outer boundary layer. They return as an oblique wave so the process of compression and expansion begins again. The end result is that a second Mach Disk will be formed where the compressed flow from the oblique shock becomes parallel with the flow and a normal shock wave forms. One other important feature of the flow is that immediately behind the Mach Disk the flow momentarily will be sub-sonic but quickly returns to supersonic.

The Mach Disk will evolve into a diamond shape under the complex flow of a rocket exhaust and becomes visible when the resultant temperature and pressure ignite unburned fuel. The area between the nozzle exit and the first diamond is known as “zone of silence” and its distance from the end of the nozzle can be calculated roughly using the following formula.

\[ x = 0.67D_0 \sqrt{\frac{P_0}{P_1}} \]

x = zone of silence distance
P0 = flow pressure
P1 = atmospheric pressure
D0 = nozzle diameter

The pattern of disks would continue to repeat indefinitely if the gases were thermodynamically ideal and frictionless, however turbulent flow at the boundary between the plume and the atmosphere (known as the contact discontinuity) will dissipate the wave pattern with distance.

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Finally, I promised to discuss a subject that is much closer to home for model rocketry fans, the chemistry behind the colors in your solid propellant model rockets exhaust. We are talking about non-black powder solid propellants. These rocket motors deliver 2 to 3 times the impulse of conventional black powder propellants. But what produces the red, blue, green or white colors in their exhaust?

When atoms are excited during combustion and the electrons return to their ground state they will emit a photon of visible light at various wavelengths and color. We will discuss what are the most likely molecular compounds used to produce these appealing exhaust emissions.

Although the formulas for these motors are proprietary, hints to their composition can be gained from their Material Safety Data Sheets (MSDS) which are required to be publically available. The MSDS states that there are products that contain varying percentages of ammonium perchlorate, strontium and/or barium nitrates dispersed in a synthetic rubber. The propellant, oxidizers and colorants are mixed together with this resin. This resin is most likely hydroxyl terminated polybutadiene (HTPB) that has been reacted with an isocyanate like methylene diphenyl di-isocyanate (MDI). The HTPB has two reactive hydroxyl groups (OH) on each end of the individual polymer units. The length of these polymers will dictate how rubbery the mixture becomes after it is reacted with the MDI. When a hydroxyl terminated polyol (HTPB) reacts with an isocyanate a urethane is the end result. You may be familiar with other urethanes that are used to make skateboard wheels, foam cushions and rigid foam insulation. But let’s get back on point. The specific color produced in the exhaust is dependent on the molecules that are added to the rubbery resin.

White exhaust is most likely produced when fine grained aluminum powder is mixed with the HTPB/MDI. The aluminum is also a very good fuel when mixed with a suitable oxidizer like ammonium perchlorate (NH4ClO4.) The subscript 4 behind the oxygen atom (O) indicates that there are 4 oxygen atoms for every NH4ClO4 molecule that are available to support the fuel decomposition reaction.

The red color in the exhaust is likely produced by the addition of Strontium Nitrate (Sr(NO3)2) to the rubbery resin. This salt of Strontium is also a very powerful accelerant since it contribute 2 X 3 oxygen atoms for a total of 6 oxygens/molecule. Strontium nitrate is a common ingredient in summer fireworks displays.

Green colors are likely produced using Barium Nitrate (Ba(NO3)2) or Barium Chloride (BaCl2) as the ingredient. There are also 6 oxygen atoms present in this molecule to help with the oxidizing of fuel. Salts of barium like barium chloride are highly toxic. Even though it...
is likely only used sparingly to produce the green exhaust it is a good safety practice to have spectators and yourself stand upwind when launching rockets with this engine. It is also unstable at room temperature so a chlorinated butadiene rubber may also be in the this fuel.

Finally, the blue color is one of the most difficult to produce. Copper compounds are the best to produce a blue exhaust plume. Paris green or copper triarsenite is a very toxic poison and it’s doubtful that it’s being used in rocket propellants. Copper chloride will produce a blue flame but it’s unstable in a hot environment like rocket exhaust. Copper nitrate (Cu(NO₃)₂) may be added and it has the bonus of adding 6 oxygens atoms for combustion. Copper carbonate (CuCO₃) in combination with ammonium perchlorate will also produce a blue exhaust. Blue is without a doubt the most difficult to reproduce and not without some amount of controversy as there have been some links to polychlorinated dioxins (a potent carcinogen) forming as reaction products.

Aluminum and magnesium alloys known as magnalium may be used in all of these formulations because it generates a very hot reaction without washing the colors out.

One final precautionary note. All propellants give off varying amounts of hydrogen chloride (hydrochloric acid) and carbon monoxide. Disposable rubber gloves are recommended when handling the left over propellant casings after launch.

I hope you enjoyed reading about some of the things going on in rocket or fighter jet exhausts. Watching NASA or Space X launches will be more meaningful knowing what the color and shape of their respective exhaust plumes look like. If you’re interested in more details I have listed some of the references used to write this paper below.

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