PEAK OF FLIGHT

NEWSLETTER

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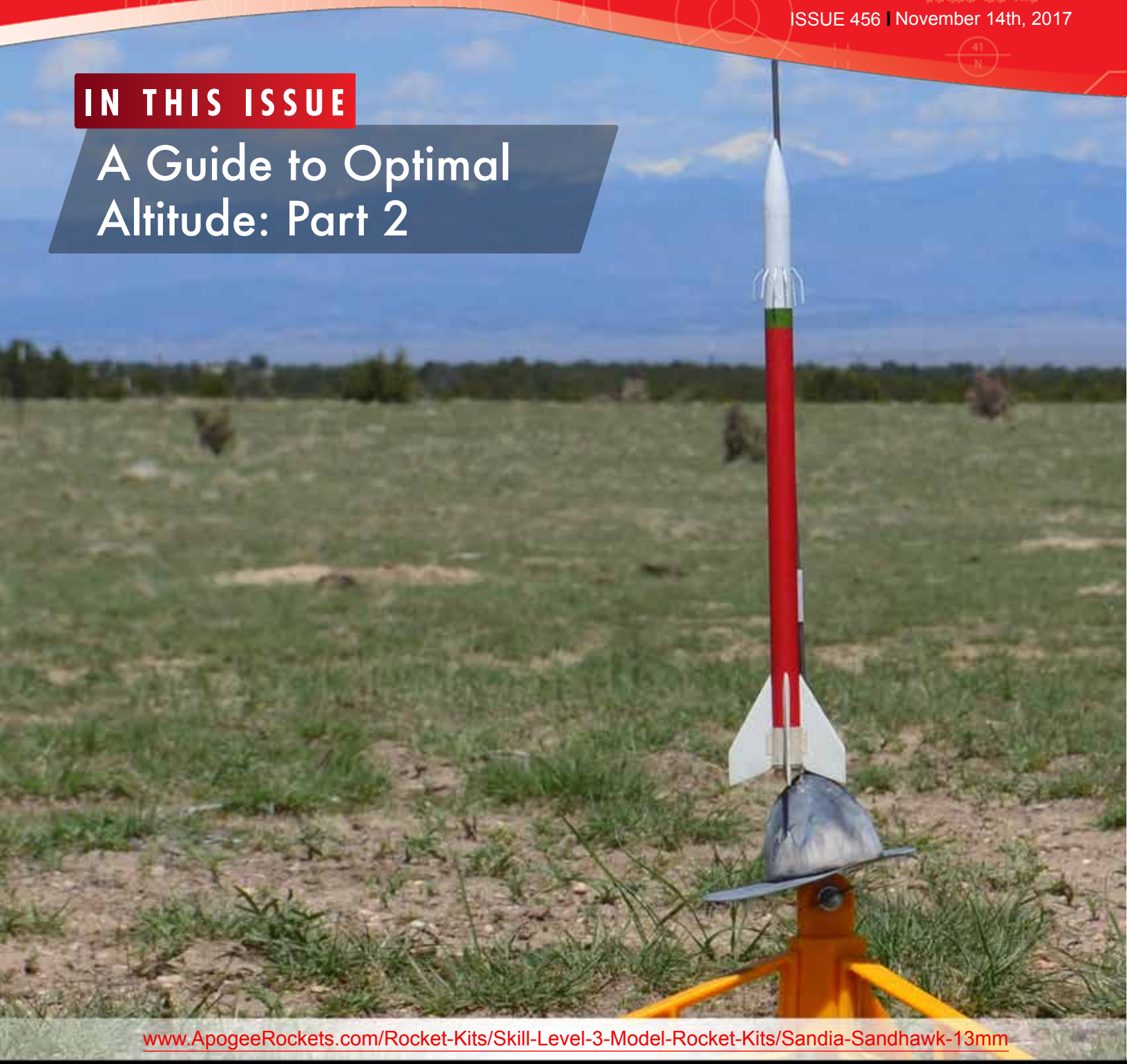

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PEAK OF FLIGHT

A Guide to Optimal Altitude: Part 2

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By Steve Ainsworth

Chapter 3: Rocket Fundimentals

This chapter provides some equations that we will need later and is a good review of the basics. I recently read a book titled The Exploration of Space by Arthur C. Clarke (published in 1951) which provided a determination of the maximum velocity possible utilizing a rocket motor when starting from earth. Since Clarke is one of my favorite authors, I bought a used hardbound copy from Amazon books for just over $5! When the book arrived, I remembered reading it as a teen from the school library, and relearned some rocket fundamentals.

Throwing Bricks From a Trolley Car

The analysis presented by Clarke is very straightforward and lends itself to an intuitive understanding of rocket fundamentals without having to gnaw through difficult math concepts. As a matter of fact, Clarke's analysis helped me understand several non-intuitive (for me at least) concepts of rocketry like Specific Impulse (Isp) which has units of just seconds, and why a high exhaust velocity for a rocket motor is a very good thing. The mathematical analysis itself originated with Konstantin Tsiolkovsky. Pavel Checkov would be proud.

The thrust of a rocket is a result of the conservation of momentum and a momentum exchange between the rocket and the expelled propellant.

Consider a man standing on a frictionless (to keep it simple) railway trolley car that has a stack of ten pound bricks on it. In the following discussion, "trolley" will mean the trolley car plus the man. When the man throws a brick off the back of the trolley, the trolley moves in the opposite direction in accordance with the conservation of momentum. The momentum added to the brick by the man throwing it rearward, is added to a forward motion of the trolley. The math (see **Table 3-1** for unit conversions) goes like this:

Propellant Momentum P (kg-m/s) = m (kg) *V (m/s)

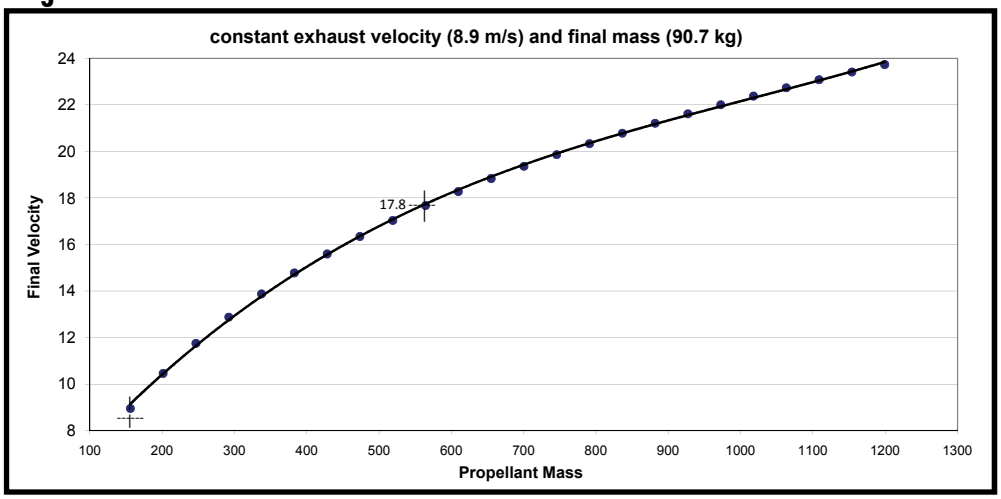
Where m is the mass of the propellant exhausted (one brick is ten pounds) 4.54 kg, and V is the velocity of the throw (Clarke assumed 20 mph) 8.94 m/s.

Lets assume the mass of the trolley is 200lbs (90.7 kg). What mass of bricks must be carried for the final velocity of the trolley to be equal to the throwing velocity (8.9 m/s)? If your first guess is 200 lbs, consider that the first brick thrown must accelerate not just the trolley but also the other bricks. We need a total brick mass equal to 1.7 times the mass of the trolley or 344 lbs (156 kg) of bricks (**Figure 3-1**, data point 1).

Table 3-1

1	2	3	4	5	6	7	8
	--- Basic Units ---						
	Time	Length	Mass	gravity	velocity	force	
Metric mks	1.0	0.305	4.536	9.807	8.940	1,431.10	32.19
Symbol	s	m	kg	g	v	nt (or N)	v
Units	second	meter	kilogram	m/s/s	m/s	newton	km/hr
Basic Units				m/s^2	m/s	kg-m/s^2	
To English	* 1.0	* 3.281	* 2.205	* 3.281	* 3.281	* 0.225	* 0.621
English fps	1.0	1.0	10.0	32.17	29.3	321.7	20.0
Symbol	s	f	lbm (or lb)	g	v	lbf (or lb)	v
Units	second	feet	pounds	f/s/s	f/s	pounds	miles/hr
Basic Units				f/s^2	f/s	lbm-f/s^2	
To Metric	* 1.0	* 0.305	* 0.454	* 0.305	* 0.305	* 4.448	* 1.61

Figure 3-1



If 344 lbs of bricks are thrown off at 20 mph (8.94 m/s), then the velocity of the trolley after the last brick is thrown off will be 20 mph. What throwing velocity is needed to double the final velocity of the trolley? Intuition says the throwing velocity must double. That is correct. If 344 lbs of bricks are thrown off at 40 mph (17.9 m/s) then the velocity of the trolley after the final brick is thrown off will be 40mph (**Figure 3-2, Page 3**).

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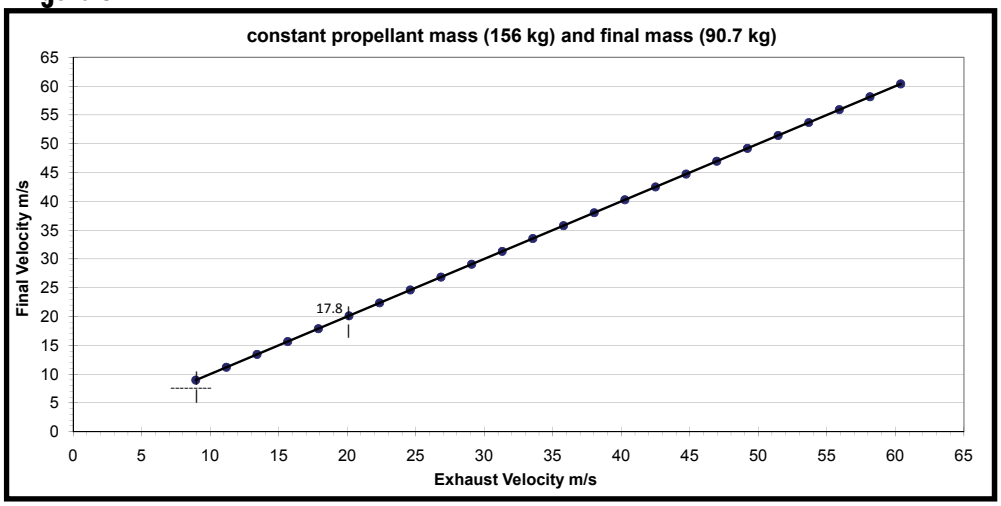
Can the man get the trolley going faster by adding more bricks instead of throwing them faster? Yes, but to double the final velocity we must have a mass of bricks equal to 6.4 times the mass of the trolley (**Figure 3-1, Page 2**). That is, for the final velocity of the trolley to double with the same throw (exhaust) velocity, we need to have 1,280 lbs (580.6 kg) of bricks.

In order to double the final velocity, we can either increase the propellant exhaust velocity by a factor of 2.0 or increase the propellant mass by a factor of 6.4. The exhaust velocity for a typical high powered rocketry motor is on the order of 6,000 f/s (1,829 m/s or 4,091 mph). The more energetic liquid-fueled motors can provide an exhaust velocity of 7,000 f/s (2,134 m/s or 4,773 mph) (Ref 9). The velocity required to achieve orbit is on the order of 26,400 f/s (8,047 m/s or 18,000 mph).

Figure 3-2



Specific Impulse

People that design rocket motors use what is termed Specific Impulse (Isp) as a performance parameter to compare one motor system with another. Isp is the thrust force as a function of the propellant mass flow rate (m) kg/s.

In the following equation:

g = gravity acceleration = 32.2 f/ s2 = 9.807 m/s2

F = Thrust magnitude nt (kg-m/s2)

Isp = F/(m * g) = Specific Impulse [kg-m/s2]/[(kg/s)*(m/s2)] = seconds

Isp is defined with units of seconds so that is has the same value regardless of the units system used for its calculation. So Isp in Metric or English units will have the same value.

The thrust over mass flow rate is divided by g to eliminate the mass and length units so that Isp will be the same in both English and Metric units. The result is that the units for Isp is seconds.

For rocket motors **Equation 3.1:, F/m = Ve (1)**. Where Ve is the exhaust velocity, or for our trolley, the velocity of the thrown bricks. So we get: **Equation 3.2 : Isp = Ve/g**.

Isp is a function of exhaust velocity. The higher the exhaust velocity, the higher (and better) the Isp. Clarke's discussion above shows why a high exhaust velocity is VERY desirable for rocket motors.

Ten Pound Brick Propellant

Lets take a look at Clarke's brick propelled trolley again. For the 20 mph (= 29.3 f/s) exhaust velocity, the Isp = [29.3 f/s]/[32.2 f/s2] = 0.91 seconds. If we could get the guy to throw 100 mph bricks, the Isp would be 4.6. Table 3-2 shows how it compares to actual rockets.

Looks like the brick propellant system is a real brick. To do or not to do the math Clarke did not do the math. The math requires just a bit of calculus. For the record, here's the math. You can skip down to **Equation 3.3 (Page 4)**, the Ideal Rocket Equation, if you would like.

Continued on page 4

Gyro Chaser Helicopter Rocket

- Unique 'transforming' rocket - looks like a normal rocket, but then rotor blades pop out at ejection
- Competition efficiency: high flights and long descent time
- Features curved rotor blades and free-spinning hub, just like those used in international competitions
- Versatile: can use any 18mm diameter motor
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Momentum = mv; mass times velocity (definition)
dm = infinitesimal (almost zero) propellant mass to be exhausted
m = mass of the vehicle including propellant mass dm (kg)
Mi = mass initial = m (kg)
Mf = mass final = (m-dm)
m = mass flow rate (kg/s)
dv = infinitesimal change in velocity of the vehicle
Ve = exhaust velocity of the propellant mass (how fast a brick is thrown) (m/s)
Delta v = change in velocity of the vehicle (m/s)
ln = natural log

(m-dm)dv = -dm*Ve (momentum of the vehicle equals the momentum of the propellant; conservation of momentum)
= mdv + dm*dv = -dm*Ve

Given; dm* dv = 0
Then: mdv = -dm*Ve, solving for dv; dv = -Ve * dm/m and after integrating

Delta v = -Ve*(ln(Mf) – ln(Mi)) = -Ve*ln(Mf/Mi)

Equation 3.3 : Delta v = -Ve*ln(Mf/Mi);
This is the formula Clarke used to get the mass of bricks required for a given final velocity and a given exhaust velocity. It is known as the Ideal Rocket Equation.

Equation 3.4: F = m * Ve (kg-m/ s^2)
Actually F/m = effective exhaust velocity which differs from our use of Ve for the brick example slightly. For an actual rocket motor the effective exhaust velocity is equal to the exhaust velocity plus the pressure thrust and reduced by the nozzle efficiency.

Table 3-2 ISP Values

	Isp	
Type	low	high
Brick	0.9	4.6
Cold Gas	60	250
Liquid	140	460
Solid	160	300
Hybrid	160	350
Nuclear	800	6,000
Electric	500	10,000

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Chapter 4: Optimal Velocity

Most hobbyists are aware that there is an optimum weight for a rocket with a given motor that will cause the rocket to achieve the greatest altitude. Some flight simulation software will even calculate that optimum weight for you. Is there also an "optimum velocity" that will provide the greatest altitude for a given motor impulse and airframe? Put simply, yes. But it is not that simple. The "optimum velocity" varies with altitude. It is time to go metric. English units have a problem for rocketry use. In the everyday world we refer to mass by the term lbs. In English units, the term for mass is actually Slugs (or even lbs. mass vs. lbs. force), which is lbs./g (g = force of gravity). This gets very confusing. In the mks (meters kilograms seconds) metric system, force is newtons (n) and mass is kilograms (kg). I have included a conversion table (**Table 3-1, Page 2**) for those that still want the final numbers in English units.

Lets Review a Bit

If a rocket has an empty mass of 20kg, what mass of propellant is required for it to achieve
a final velocity of 1000m/s (2237mph) if the exhaust velocity for the motor is 2000m/s?

From the Ideal Rocket Equation (**Equation 3.3, Page 4** and **Equation 4.1** below), we find that the rocket would need 0.65 * 20kg or 13kg of propellant with an exhaust velocity of 1000 m/s to achieve that final velocity. Recall that for an exhaust velocity of 2000m/s, the propellant would have an Isp of (2000 m/s) / (9.8 m/s2) = 203.9 seconds. This would be a motor with a total impulse of 26,000 n-s (13kg * 2000 m/s).

Equation 4.1: Ideal Rocket Equation (solved for initial mass): mi = mf * e(Vf / Ve) mi = mass initial (rocket + propellant)
mf = mass final (20kg)
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Optimal Velocity 4
Vf = Final Velocity (1000 m/s) assuming a
zero initial velocity.
Ve = Exhaust Velocity (2000m/s)

This calculation assumes that the rocket is not acting against gravity or drag (it is in space). In the real world of High Powered Rocketry, neither of these assumptions is accurate. When a rocket is rising from the earth, it must overcome gravity and the drag forces of the air.

The force of gravity subtracts 9.807 m/s from the rockets vertical velocity for each second of flight! If the motor has a burntime of 20 seconds, gravity reduces the final burnout velocity by 196 m/s. If it had a burnout velocity of 1000 m/s without gravity, you would see a burnout velocity of 803 m/s (= 1000 – 20 *9.807). If the rocket had a 2 second burntime, the burnout velocity would be 980 m/s (= 1000 – 2*9.807). Therefor, the sooner you get the rocket to its maximum velocity (the shorter the motor burntime), the less time gravity has to reduce the total force of the motor slow it down.

However, shortening the burntime causes another problem to crop up. As shown above, the shorter the burntime, the higher the maximum velocity of the rocket. Unfortunately, as the rocket travels faster in the air, the aerodynamic drag force increases exponentially.

Equation 4.2: D = 0.5 * Cd * Rh *v^2 *A
D= Drag Force (Newtons)
Cd = Drag Coefficient (Usually between 0.01and 0.5) (Unitless)
Rh = Air Density (kg/m^2)
v = Rocket Velocity (m/s)
A = Reference area of the airframe (m^2)

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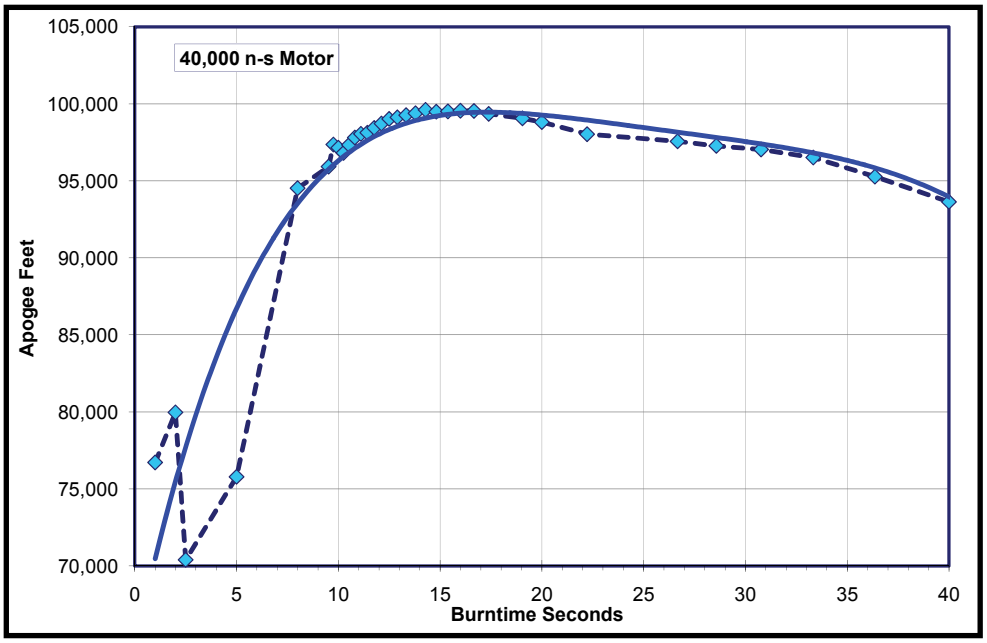
Roughly speaking, within the atmosphere, the drag force increases with the square of the velocity. If you double the velocity, the drag force increases by a factor of 4!

The longer the bur time, the greater the impact of gravity (and gravity turning).

The shorter the burn time, the greater the impact of drag.

The logical question is for a given rocket, what motor burntime (average thrust) will yield the highest altitude? I ran about 20 RockSim simulations using the same airframe and total impulse motor with a range of burntimes, to see if there would be an optimum burntime. Optimum being the burntime that provides the highest altitude for a given total impulse. The results of those runs are provided in **Figure 4-1**.

Figure 4-1



Typically with rocketry, we are trying to get the highest altitude possible from a particular total impulse motor and airframe design. So, if you could have any thrust curve that you wanted, what thrust curve would provide the highest altitude for a given airframe? To find this out, we need to determine the optimum velocity for that airframe as it ascends.

We need to calculate the gravity losses and add those to the drag losses for a range of velocities at each altitude increment. Then by inspection, we can discover the velocity that will yield the smallest losses, and thus the highest altitude. This information together with the airframe design data will allow us to calculate the thrust required at each interval, giving us the optimum thrust curve.

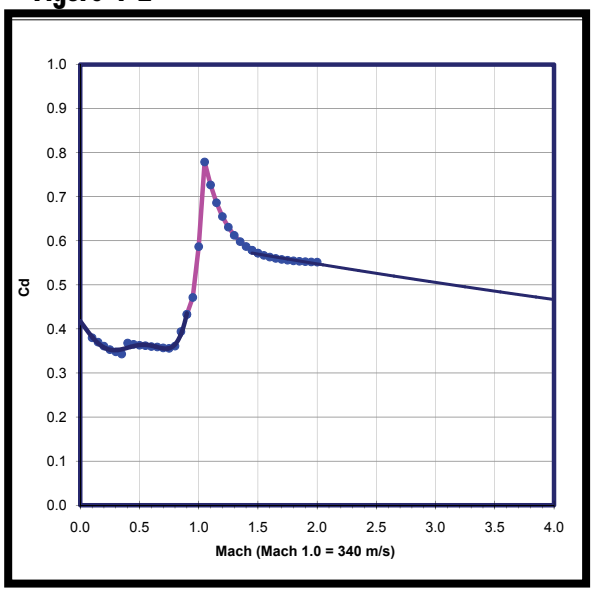
Equation 4.3: Lgv = 9.807 m/s2 * Time = Lgv m/s = Velocity loss due to gravity

The gravity loss is just 9.807 m/s/s times time, which provides the velocity loss m/s. For this analysis, we are using an altitude increment of 1,000 meters. The time required for the rocket to travel that increment is = 1000m / Vm/s = 1,000/V seconds.

The drag loss is provided by **Equation 4.2** (Page 5). The drag calculation requires that we know the drag coefficient Cd. The problem here is that Cd is not a constant (**Figure 4-2**). It varies with the rocket airspeed as a function of the Mach number (Mach 1 = the speed of sound = 340m/s at ground level with a standard atmosphere). The actual airspeed of Mach 1 varies with altitude (**Figure 4-3, Page 7**), as its a function of air density and temperature.

The drag loss calculation also requires that we know the air density (Rh), and Rh varies with altitude. We have too many things varying to make it nice and neat. My approach is to run the calculations at specific increments of velocity and altitude and add gravity losses to drag losses to find the velocity that provides the minimum total losses.

Figure 4-2



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Another approach is to simply use RockSim and run a range of burn-times.

Figure 4-4 shows the gravity loss, drag loss and summed losses for an airframe that consists of a booster connected to a second stage (sustainer) for a range of velocities at a specific altitude (5,000 meters). **Figure 4-5** provides the data for the Sustainer at 20,000 meters. After this data is plotted for altitude increments up through 40,000 meters, and the optimum velocity is picked out for each altitude increment we get **Figure 4-6**, which now includes the optimum velocities for both the booster with the sustainer, and the sustainer by itself.

This graph confirms the earlier finding that long burn motors providing slower rockets go higher (as long as they go straight up). But it also tells us just what velocity is best. Now, if you run these calculations for your very sleek minimum diameter rocket, find a motor that will provide velocities near the optimum, launch on a calm day and are lucky enough to achieve a near vertical flight, then you can achieve the maximum possible altitude using that airframe and total impulse.

These graphs are based on the to 100k rocket project described in Chapter 5.

We will discuss more on this topic in the upcoming Peak of Flight Newsletter "A Guide to Optimal Altitude: Part 3"

Figure 4-3

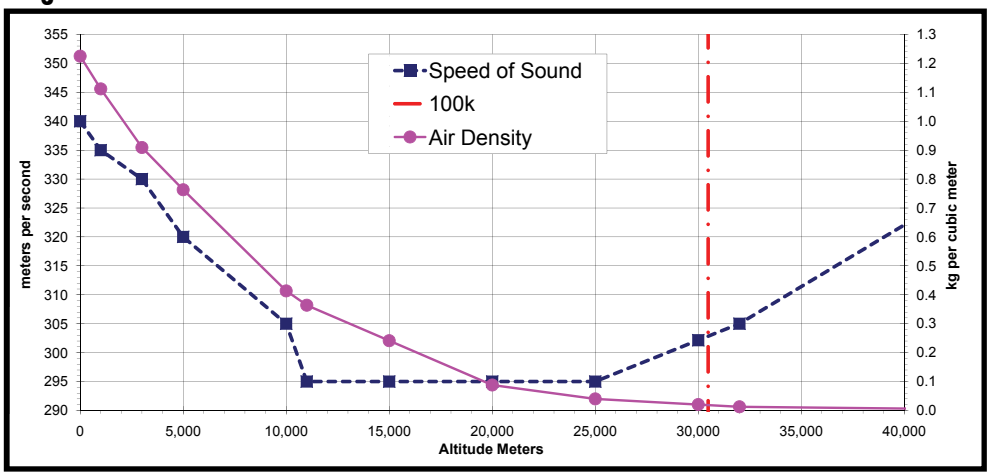
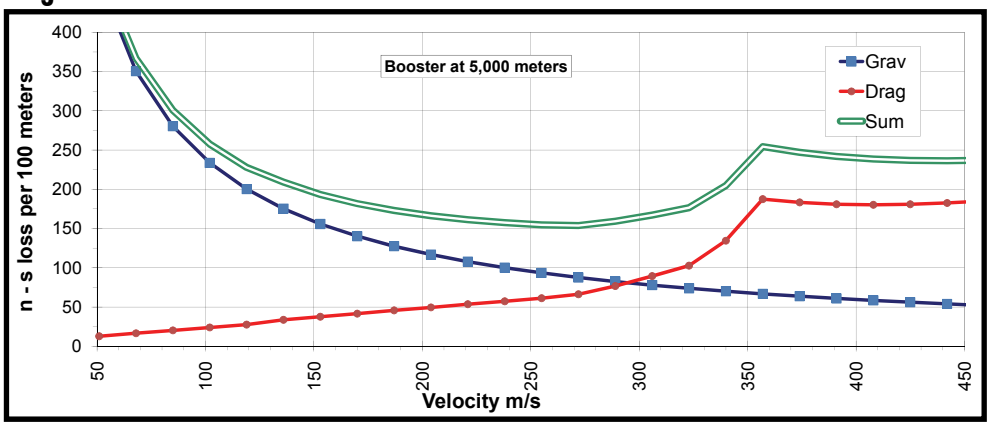


Figure 4-4



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About The Author:

Steve Ainsworth is a Civil Engineer with a Master's degree in Physics from UNLV. He has published articles in both HPR and ER since he became a "Born Again Rocketeer" in 1994. Steve joined Tripoli Vegas in 1995 and began flying his "gizmo" rockets. In 1995, Las Vegas was the perfect setting for re-entering rocketry with Gary Rosenfield of Aerotech providing motor tutoring and Tom Blazinin (TRA # 003) as the local Prefect, coaching and signing for Steve's Level 1 and Level 2 flights. While on an extended engineering assignment in northern California, Steve had evening time to think about rocketry. He used that time to learn enough aerodynamics to develop a computational method for flight predictions based on Gordon Mandell's MIT Thesis articles.

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Figure 4-5

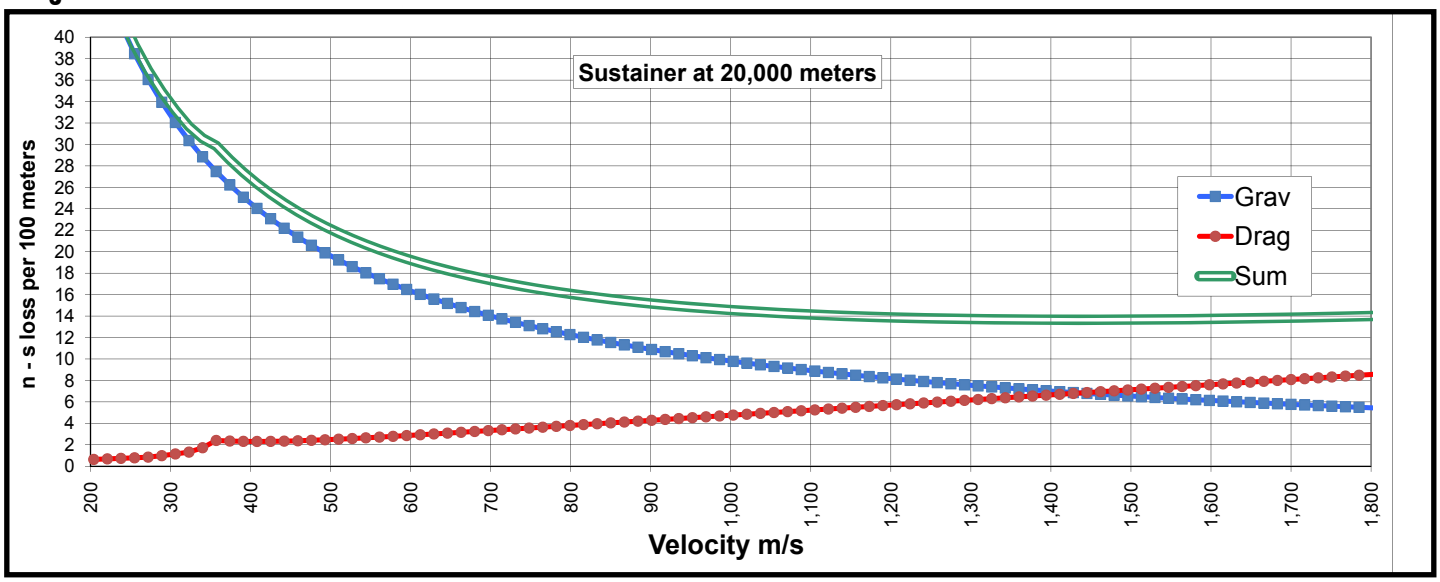
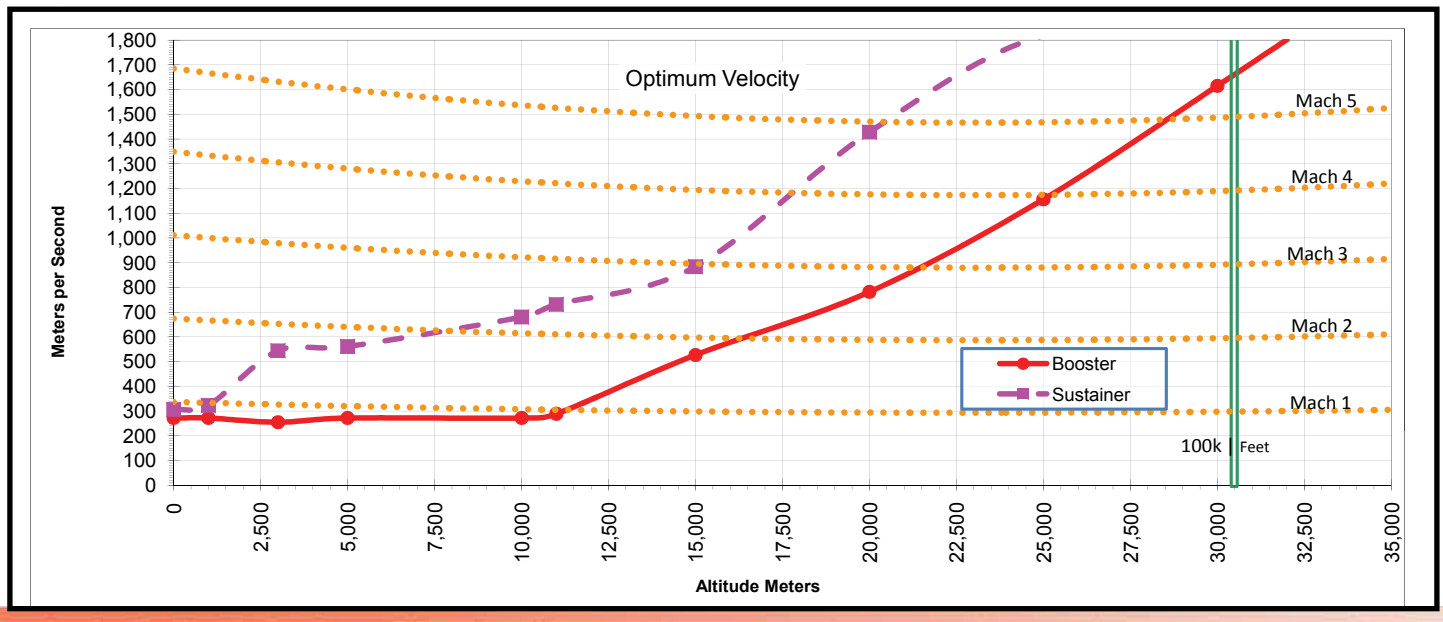
Figure 4-6