PEAK OF FLIGHT

NEWSLETTER

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PEAK OF FLIGHT

A Guide to Optimal Altitude: Part 3

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By Steve Ainsworth

Chapter 4: Optimal Velocity

Most hobbyists are aware that there is an optimum weight for a rocket with a given motor that will cause the rocket to achieve the greatest altitude. Some flight simulation software will even calculate that optimum weight for you. Is there also an "optimum velocity" that will provide the greatest altitude for a given motor impulse and airframe? Put simply, yes. But it is not that simple. The "optimum velocity" varies with altitude.

It is time to go metric. English units have a problem for rocketry use. In the everyday world we refer to mass by the term lbs. In English units, the term for mass is actually Slugs (or even lbs. mass vs. lbs. force), which is lbs./g (g = force of gravity). This gets very confusing. In the mks (meters kilograms seconds) metric system, force is newtons (n) and mass is kilograms (kg). I have included a conversion table (**Table 3-1, Page 3**) for those that still want the final numbers in English units.

Lets Review a Bit

If a rocket has an empty mass of 20kg, what mass of propellant is required for it to achieve a final velocity of 1000m/s (2237mph) if the exhaust velocity for the motor is 2000m/s?

From the Ideal Rocket Equation (**Equation 3.3** and **Equation 4.1** below), we find that the rocket would need 0.65 * 20kg or 13kg of propellant with an exhaust velocity of 1000 m/s to achieve that final velocity. Recall that for an exhaust velocity of 2000m/s, the propellant would have an Isp of (2000 m/s) / (9.8 m/s^2) = 203.9 seconds. This would be a motor with a total impulse of 26,000 n-s (13kg * 2000 m/s).

Equation 3.3 : Delta v = -Ve*ln(Mf/Mi);
Equation 4.1: Ideal Rocket Equation (solved for initial mass): $mi = mf * e^{(Vf / Ve)}$

mi = mass initial (rocket + propellant)
mf = mass final (20kg)
Vf = Final Velocity (1000 m/s) assuming a zero initial velocity.
Ve = Exhaust Velocity (2000m/s)

This calculation assumes that the rocket is not acting against gravity or drag (it is in space). In the real world of HPR, neither of these assumptions is accurate. When a rocket is rising from the earth, it must overcome gravity and the drag forces of the air.

The force of gravity subtracts 9.807 m/s from the rockets vertical velocity for each second of flight! If the motor has a burntime of 20 seconds, gravity reduces the final burnout velocity by 196 m/s. If it had a burnout velocity of 1000 m/s without gravity, you would see a burnout velocity of 803 m/s (= 1000 – 20 *9.807). If the rocket had a 2 second burntime, the burnout velocity would be 980 m/s (= 1000– 2*9.807). Therefore, the sooner you get the rocket to its maximum velocity (the shorter the motor burntime), the less time gravity has to reduce the total force of the motor slow it down.

However, shortening the burntime causes another problem to crop up. As shown above, the shorter the burntime, the higher the maximum velocity of the rocket. Unfortunately, as the rocket travels faster in the air, the aerodynamic drag force increases exponentially.

Equation 4.2: $D = 0.5 * Cd * Rh * v^2 * A$
D= Drag Force (Newtons)
Cd = Drag Coefficient (Usually between 0.01 and 0.5) (Unitless)
Rh = Air Density (kg/m^2)
v = Rocket Velocity (m/s)
A = Reference area of the airframe (m^2)

Roughly speaking, within the atmosphere, the drag force increases with the square of the velocity. If you double the velocity, the drag force increases by a factor of 4!

The longer the burntime, the greater the impact of gravity (and gravity turning).

The shorter the burntime, the greater the impact of drag. The logical question is for a given rocket, what motor burntime (average thrust) will yield the highest altitude? I ran about 20 RockSim simulations using the same airframe and total impulse motor with a range of burntimes, to see if there would be an optimum burntime. Optimum being the burntime that provides the highest altitude for a given total impulse. The results of those runs are provided in **Figure 4-1, Page 3**.

Typically with rocketry, we are trying to get the highest altitude possible from a particular total impulse motor and airframe design. So, if you could have any thrust curve that you wanted, what thrust curve would provide the highest altitude for a given airframe? To find this out, we need to determine the optimum velocity for that airframe as it ascends.

We need to calculate the gravity losses and add those to the drag losses for a range of velocities at each altitude increment. Then, by inspection, we can discover the velocity that will yield the smallest losses, and thus the highest altitude. This information together with the airframe design data will allow us to calculate the thrust required at each interval, giving us the optimum thrust curve.

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A Guide to Optimal Altitude: Part 3

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Equation 4.3: Lgv = 9.807 m/s^2 * Time = Lgv m/s = Velocity loss due to gravity.

The gravity loss is just 9.807 m/s/s times time, which provides the velocity loss m/s. For this analysis, we are using an altitude increment of 1,000 meters. The time required for the rocket to travel that increment is = 1000m /Vm/s = 1,000/V seconds.

The drag loss is provided by **Equation 4.2 (Page 2)**. The drag calculation requires that we know the drag coefficient Cd. The problem here is that Cd is not a constant (**Figure 4-2**). It varies with the rocket airspeed as a function of the Mach number (Mach 1 = the speed of sound = 340m/s at ground level with a standard atmosphere). The actual airspeed of Mach 1 varies with altitude (**Figure 4-3**), as it is a function of air density and temperature.

The drag loss calculation also requires that we know the air density (Rh), and Rh varies with altitude. We have too many things varying to make it nice and neat. My approach is to run the calculations at specific increments of velocity and altitude and add gravity losses to drag losses to find the velocity that provides the minimum total losses. Another approach is to simply use RockSim and run a range of burntimes.

Figure 4-4 (Page 5) shows the gravity loss, drag loss and summed losses for an airframe that consists of a booster connected to a second stage (sustainer) for a range of velocities at a specific altitude (5,000 meters).

Table 3-1

1	2	3	4	5	6	7	8
	--- Basic Units ---						
	Time	Length	Mass	gravity	velocity	force	
Metric mks	1.0	0.305	4.536	9.807	8.940	1,431.10	32.19
Symbol	s	m	kg	g	v	nt (or N)	v
Units	second	meter	kilogram	m/s/s	m/s	newton	km/hr
Basic Units				m/s^2	m/s	kg-m/s^2	
To English	* 1.0	* 3.281	* 2.205	* 3.281	* 3.281	* 0.225	* 0.621
English fps	1.0	1.0	10.0	32.17	29.3	321.7	20.0
Symbol	s	f	lbm (or lb)	g	v	lbf (or lb)	v
Units	second	feet	pounds	f/s/s	f/s	pounds	miles/hr
Basic Units				f/s^2	f/s	lbm-f/s^2	
To Metric	* 1.0	* 0.305	* 0.454	* 0.305	* 0.305	* 4.448	* 1.61

Table 4-2

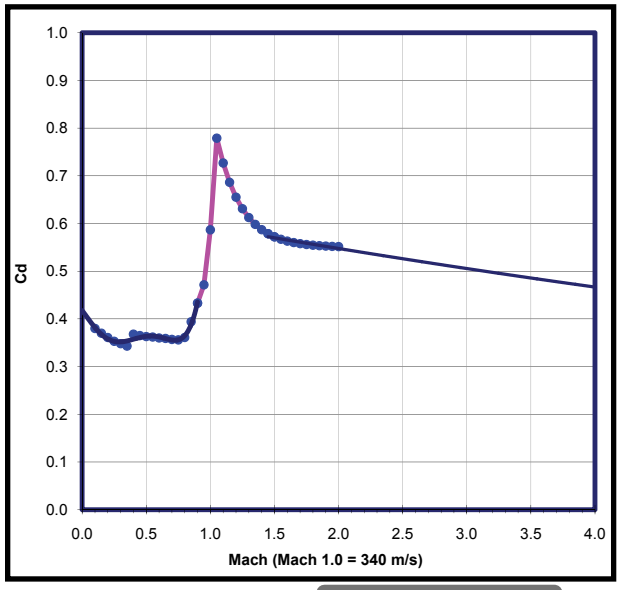
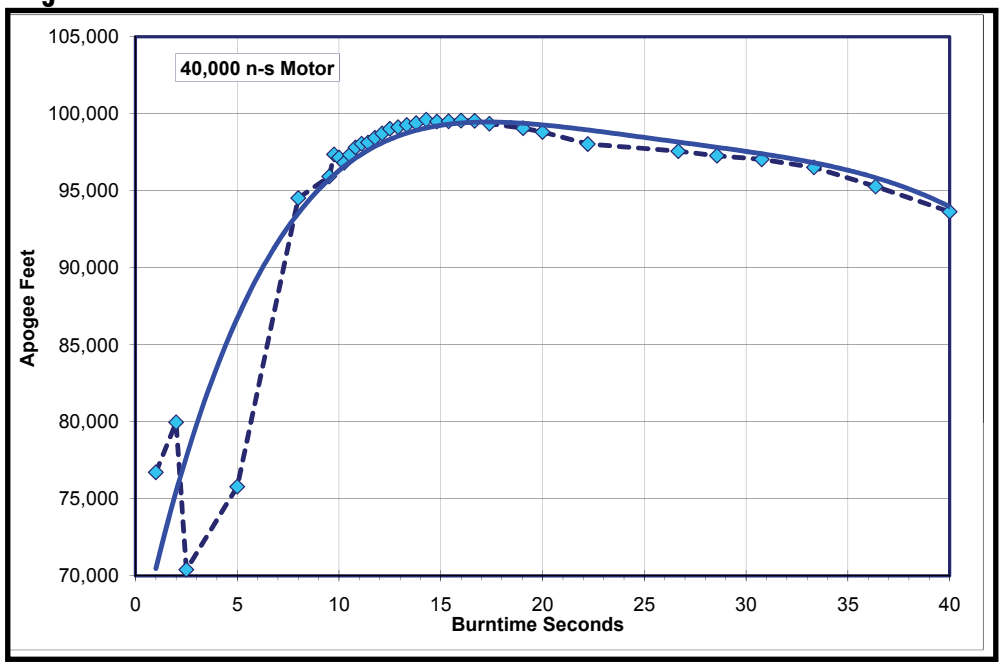


Figure 4-1



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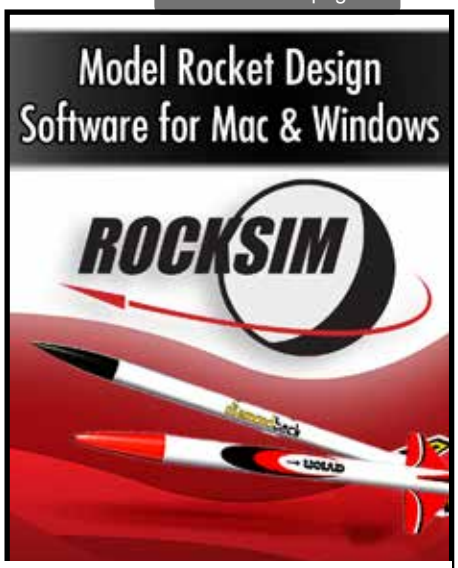

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Figure 4-5 (Page 5) provides the data for the Sustainer at 20,000 meters. After this data is plotted for altitude increments up through 40,000 meters, and the optimum velocity is picked out for each altitude increment we get **Figure 4-6 (Page 5)**, which now includes the optimum velocities for both the booster with the sustainer, and the sustainer by itself.

This graph confirms the earlier finding that long burn motors providing slower rockets go higher (as long as they go straight up). But it also tells us just what velocity is best. Now, if you run these calculations for your very sleek minimum diameter rocket, find a motor that will provide velocities near the optimum, launch on a calm day and are lucky enough to achieve a near vertical flight, then you can achieve the maximum possible altitude using that airframe and total impulse. These graphs are based on the to 100k rocket project described in Chapter 5.

Chapter 5: Optimal Thrust

As our rocket ascends, the forces that cause the rocket to slow down are drag and gravity. Assuming the rocket is on a vertical trajectory, the maximum altitude is determined by the motor thrust, drag and gravity. If we have done everything we can to reduce the atmospheric drag, and to reduce the rocket weight, then what else can be done to get a higher apogee with a given size (total impulse) motor?

Drag Losses

Last time we found:

Equation 4.2: $D = 0.5 * Cd * Rh * v^2 * A$ (newtons)

Where:
D= Drag Force (kg – m/s^2 aka: newtons; or just n)
Cd = Drag Coefficient (Usually between 0.01 and 0.5) (Unitless)
Rh = Air Density (kg/m2)
v = Rocket Velocity (m/s)
A = Reference area of the airframe (m2)

We saw that the drag losses increase with the square of the velocity. If it takes t seconds for the rocket to travel from altitude h1 to altitude h2, then:

v (m/s) = $(h2 – h1)/t$

We can plug this value for v into **Equation 4.2** and compute the resulting drag losses in newtons. Multiplying **Equation 4.2** by the time step t we will get newton – seconds of drag losses:

Equation 5.1a: $Di = 0.5 * Cd * Rh * [(h2 – h1)/t]^2 * A * t$ (newton – seconds)
Or: $Di = 0.5 * Cd * Rh * [(h2 – h1)]^2 * A / t$ (newton – seconds)

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Continued from page 4

Where Di is the Drag Impulse in newton –seconds (like a motor).

With this equation we can determine how much of the motor impulse is consumed by drag forces. The shorter the motor burn time, the higher the maximum velocity of the rocket. The higher the maximum velocity of the rocket, the greater the drag. And more drag means more of the motor impulse will be utilized overcoming drag forces. So, slower is better for drag.

Gravity Losses

To compute the losses due to gravity, we now add:

Equation 5.2: Fg = Mrt * g = Mrt * 9.807 (n)

Where:

Fg = gravitational force (newtons aka: n)

Mrt = total mass of the rocket (kg)

g = acceleration due to gravity = 9.807 m/s^2

Again, if it takes time = t for the rocket to travel from altitude h1 to altitude h2, then multiplying the results of equation 5.2 by that time step t we get:

Equation 5.2a: Fgi = Mrt * g * t (newton – seconds)

Where Fgi is the Gravity Force Impulse in newton – seconds (like a motor).

With this equation, we can determine how much of the motor impulse is consumed overcoming gravity. The shorter the motor burn time, the less time there is for gravity to slow the rocket. Less time being slowed by gravity means less total impulse used in overcoming gravity forces. So, faster is better for gravity.

Also while the motor is burning, the rocket will be impacted by gravity turning which will alter the trajectory away from vertical, and create horizontal velocity. This horizontal velocity will consume motor impulse in increasing the downrange distance and lowering the maximum altitude. Therefore, as discussed earlier, for the gravity turning component shorter burntimes equate to higher altitudes.

Total Losses

For now we will ignore gravity turning and assume a completely

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Figure 4-3

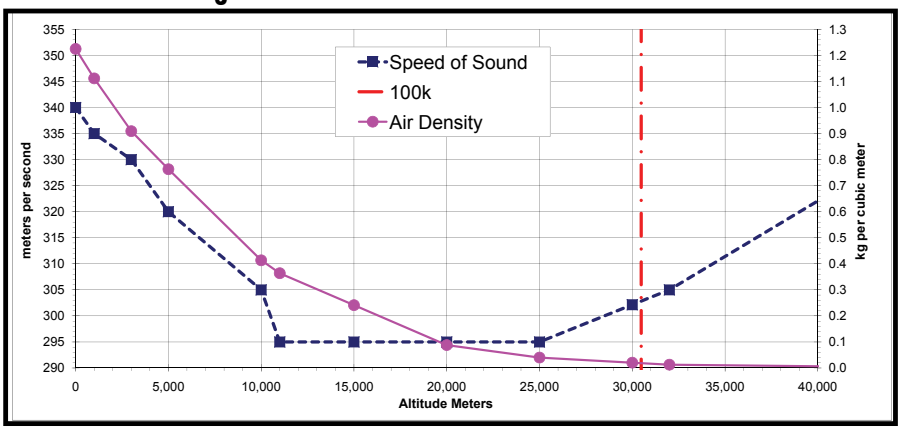


Figure 4-4

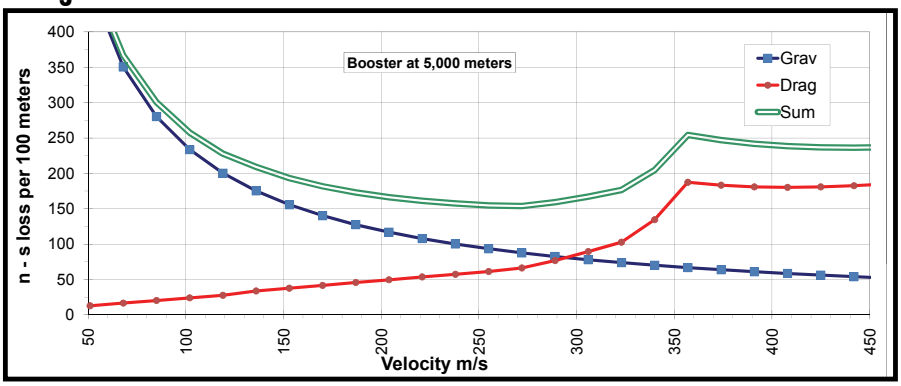


Figure 4-5

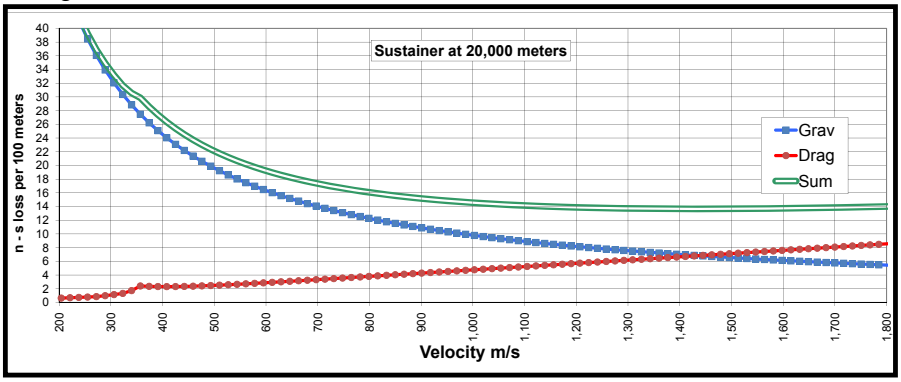
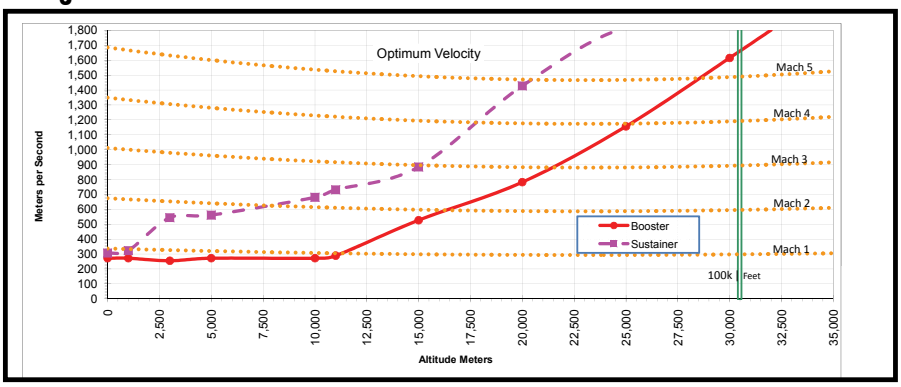


Figure 4-6



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vertical trajectory. Which of the loss forces rules, drag or gravity? Lets add the two loss equations together and run some iterative calculations to find the minimum total. By adding **Equation 5.1 (Page 4)** and **Equation 5.2a (Page 5)** we get the Impulse Loss Total (Ilt) due to the combined gravity and drag forces:

Equation 5.3: Ilt = D + Fg = [0.5 * Cd * Rh *[(h2 – h1)]^2 *A / t] + [Mrt * g * t] = Impulse Loss Total

Rh varies with altitude but is assumed to be constant over time step t.

Mrt varies as the propellant is burned but is assumed to be constant over time step t.

Cd varies with velocity but is assumed to be constant over time step t.

"A" is a constant (Reference area).

"g" is a constant (Acceleration due to gravity)

(h2 – h1) is assigned by us, the smaller the value, the more accurate the solution.

"t" is a function of velocity, as how fast the rocket is going determines how long it takes to get from h1 to h2.

Drag increases as velocity increases. Gravitational losses decrease as velocity increases. One goes up with velocity, and one goes down. What we want is the velocity that will have the smallest Impulse Loss Total. If we can fly at that velocity, then we will have the maximum possible motor impulse utilized for altitude gain instead of used overcoming drag and gravity.

Sample Rocket

The rocket used for the sample calculations is shown in **Figure 5-1**. We have a 166mm diameter, 9.7 kg (dry) booster with a 29,338 n-s O motor, and a 4.3kg, 76mm diameter sustainer stage with an M3500 motor. The total weight with motors is 46.8 kg.

Time for a Spreadsheet

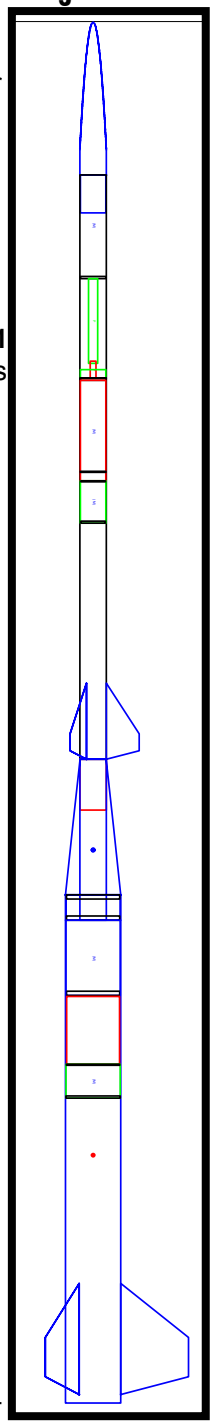
Since **Equation 5.3** is a bit difficult to solve for the optimum velocity directly, I chose to find an approximate solution by iteration using a series of spreadsheets (**Table 5-1, 5-2 Page 7**) with each row a different (and increasing) velocity. I set (h2 – h1) to 100 meters. There is one sheet for each altitude with rows with increasing velocity and columns indicating:

Mach #
Cd for the corresponding v
D for the corresponding v t (seconds required for the rocket to travel 100 meters)
D for the corresponding v
Fgi for the corresponding v
Ilt for the corresponding v

The spreadsheet has cells at the top for user input of the constant values needed for the computation.

By searching down the Total Impulse Loss (farthest right) column of each sheet for the minimum value (of Ilt), and sliding left along that row to find the velocity that corresponds to that minimum Ilt, we have the "speed to fly" for that altitude. **Table 5-1 (Page 7)** is a portion of the series of sheets for the booster with the sustainer attached, and **Table 5-2 (Page 7)** is a portion of the series of sheets for the sustainer alone. Since the booster is flying at lower altitudes in denser air, you might notice that its optimum velocity is lower than that of the sustainer alone, which begins its flight above the altitude of booster burnout.

Figure 5-2



Designer Thrust Curve

We have 29,338 newton – seconds of motor to play with on the booster. How should we spend it? For the booster (with sustainer onboard), we found that for a velocity of about 272m/s, we have minimized Ilt, the total drag and gravity losses (**Table 5-1 Page 7**). What thrust curve will yield this "speed to fly"?

We will need to incrementally work from the launch altitude and velocity to the altitude for the next time increment, find the desired velocity for that altitude and calculate the thrust necessary to produce that velocity. In this way we can build up a thrust curve for the desired velocity profile one time step at a time.

Delta ΔV

To get from one velocity to another in a given time step, we need to accelerate the rocket. The velocity relationship to acceleration and time is given by:

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Equation 5.4: vf = vi + a * t
Where:
(vf-vi) = ΔV = Change in velocity in m/s (aka Delta V)
vi = velocity (m/s), velocity at the beginning of the time step
vf = velocity (m/s), velocity at the end of the time step
a = acceleration (m/s^2), acceleration during the time step
t = time (seconds), the duration of the time step

Solving for acceleration we get:
Equation 5.4a: a = (vf-vi)/t or a = ΔV/t

Altitude is given by:
Equation 5.5: h = hi + (vi * t) + (a * t^2)

Where:
h = altitude (meters) at the end of the time step
hi = initial altitude (meters), altitude at the beginning of the time step

Now we can compute the required acceleration for the time step that will provide the optimum velocity for the rocket at the altitude flown during the time step. With this in hand, we can calculate the required motor thrust for that time step:
Equation 5.6: F= Mrt * a

Where:
F= Force in newtons (kg-m/s^2) (of the motor)
Mrt = overall mass in kg of the rocket at this time increment
a = acceleration in m/s^2

Combining **Equations 5.5** and **5.6** we get:
Equation 5.7: F = Mrt * ΔV/t

Table 5-1

10,000	m Alt	Dia =	150	Aref =	0.0177	m^2			
Density	m/s			mass =	24.3	kg			Total
0.4135	305	m/s = Mach 1		9.807	Loss	s/	Loss	Loss	Impulse
Alt	Mach		Velocity	Grav	Drag	100	Grav	Drag	Loss
Meters	#	Cd	m/s	m/s/s	kg-m/s/s	m	n - s	n - s	n - s
10,000	0.600	0.360	204	9.807	54.7	0.5	117	27	143.7
10,000	0.650	0.359	221	9.807	64.1	0.5	108	29	136.8
10,000	0.700	0.357	238	9.807	73.9	0.4	100	31	131.2
10,000	0.750	0.356	255	9.807	84.6	0.4	93	33	126.6
10,000	**0.800**	**0.361**	**272**	**9.807**	**97.6**	**0.4**	**88**	**36**	**123.5**
10,000	0.850	0.394	289	9.807	120.2	0.3	82	42	124.1
10,000	0.900	0.433	306	9.807	148.1	0.3	78	48	126.3
10,000	0.950	0.471	323	9.807	179.7	0.3	74	56	129.4

Table 5-2

10,000	m Alt			Aref =	0.001963	m^2			
Density	m/s			mass =	10.5	kg			
0.4135	305	m/s = Mach 1		9.807	Loss	s/	Loss	Loss	Impulse
Alt	Mach		Velocity	Grav	Drag	100	Grav	Drag	Loss
Meters	#	Cd	m/s	m/s/s	kg-m/s/s	m	n - s	n - s	n - s
10,000	1.650	0.560	561	9.807	71.6	0.2	18.4	12.8	31.1
10,000	1.700	0.558	578	9.807	75.6	0.2	17.8	13.1	30.9
10,000	1.750	0.556	595	9.807	79.9	0.2	17.3	13.4	30.7
10,000	1.800	0.555	612	9.807	84.3	0.2	16.8	13.8	30.6
10,000	1.850	0.553	629	9.807	88.9	0.2	16.4	14.1	30.5
10,000	1.900	0.553	646	9.807	93.6	0.2	15.9	14.5	30.4
10,000	1.950	0.552	663	9.807	98.5	0.2	15.5	14.9	30.4
10,000	**2.000**	**0.552**	**680**	**9.807**	**103.5**	**0.1**	**15.1**	**15.2**	**30.4**
10,000	2.050	0.551	697	9.807	108.7	0.1	14.8	15.6	30.4
10,000	2.100	0.551	714	9.807	114.0	0.1	14.4	16.0	30.4
10,000	2.150	0.551	731	9.807	119.5	0.1	14.1	16.3	30.4
10,000	2.200	0.550	748	9.807	125.0	0.1	13.8	16.7	30.5
10,000	2.250	0.550	765	9.807	130.8	0.1	13.5	17.1	30.6
10,000	2.300	0.550	782	9.807	136.6	0.1	13.2	17.5	30.6
10,000	2.350	0.550	799	9.807	142.6	0.1	12.9	17.8	30.7

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Propellant Burn Rate

We now know the required ΔV which together with the rocket mass specifies the required motor thrust F for each time increment, and we know the duration of each time increment. We now need to calculate the mass of the rocket (Mrt) for each time increment.

Mp = total mass of propellant
Mpi = mass of propellant at the beginning of the time increment
Mpt = mass of propellant lost during this time increment in kg.
Then: Mpf = Mpi – Mpt; mass of propellant at the end of the time increment, which will be the Mpi for the next time increment.

For simplification (and so we do not have a circular equation in Excel), we will use:
Mrt = Mpi during the entire time increment (Approximation)

For the first time increment, Mpi = Mp, and Mpf = Mpi - Mpt

For the next time increment, Mpi = Mpf from the first increment, and the new Mpf = Mpi -Mpt

Rocket Mass

The rocket mass decreases each time increment by the weight of propellant burned during that time increment (remember the bricks?). So, we need to determine the mass of propellant burned during each time increment (Mpt) that will provide the necessary thrust F.

Essentially, we need to know how much propellant will be burned in t seconds to produce the required thrust of F newtons. That is we need to know how many kg of propellant it takes to make F * t newton-seconds of thrust.

Since we know F and t, we can compute the kg of propellant required, which is Mpt. This brings us back around to Specific Impulse (Isp) and Rocket Exhaust Velocity Ve from **Chapter 3** (A Guide to Optimal Altitude Part 2). Mpt as used here is the same as the propellant flow rate (m) from **Chapter 3**, and we had:

Equation 3.1: F/m = Ve and eqn 3.2: Isp = Ve/g or
Ve = Isp/g
Where:
Ve = exhaust Velocity
g = gravity acceleration = 9.807 m/s^2
Force required = F = Ve*Mpt

And solving for Mpt:
Equation 5.8: Mpt = F/Ve = (F*g)/Isp
The higher the exhaust velocity the less propellant needed for the same thrust.

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Propellant Mass Flow Rate

The high powered rocketry hobby motor suppliers provide good information in their motor data. A motor thrust curve provides the thrust (F-newtons) over the burn time of the motor. The motor designation provides the average thrust over the burn time. Therefore; a J350 has an average thrust of 350 newtons over its burn time, with a total impulse in the J range.

The motor data sheet also provides the total propellant weight (mass) Mp.

We are looking for newton-seconds/kg of propellant which is: F/Mp.

For a CTI-O 5100 (Cesaroni O motor) we have:
Average Impulse = 5,100 newtons kg-m/s^2 (Note the motor designation: O 5100)
Total burn time = 5.75 seconds
Total Impulse Fe = 5.75 * 5100 = 29,338 newton-seconds
Propellant mass Mp = 13.245 kg

Equation 5.9: Ve = Fe/Mp = 29,339/13.245 = 2215 newton-seconds/kg; (kg-s-m/s^2)/kg; m/s

We have Ve = 2215 m/s and **Equation 3.2**: Isp = 2215/g = (2215m/s)/9.807m/s2) = 226 seconds. In this unit system Isp is the same value in mks as it is in pfs (pound – feet – seconds). The higher the Exhaust Velocity (Ve) the higher the Isp, and the more bang for the buck.

We will assume that our designer motor has the same Isp as the CTI-O 5100, and will provide 2215 newton-seconds of thrust for each kg of propellant burned. That is it will burn 1/2215 kg = 0.000451 kg of propellant for each newton-second of thrust provided.

Mp/Fe = 0.000451 kg/n-s

Time Step

If our thrust calculations show that our rocket will need 1000 newton-seconds of thrust over time increment "t" to get from the initial velocity to the optimum velocity for that altitude, we will need to burn 0.45 kg of propellant during that time increment. Thus the rocket mass will decrease by 0.45 kg. If time increment = 5 seconds, then the propellant mass flow rate is 0.45 kg/5 seconds or 0.09 kg/second.

We can then generate **Table 5-3** by selecting the velocity for each altitude that produces the smallest total losses from the sheet series in **Table 5-1**, as described above. We can then add a column providing the time step duration (Delta time) by assuming that the rocket will accelerate uniformly from the velocity at the beginning of the time step to the optimum velocity at the end of the time step. In that case the average velocity during the time step (vav) will just be one-half of the difference of the velocities at the beginning and the end of the time step.

vav = (v1 + v2) / 2

The time required is given by the change in altitude divided by velocity:

Delta time (s) = (h2 - h1)(m) / (vav)(m/s); This is the duration of this time increment.

Continued on page 10

Figure 5-2

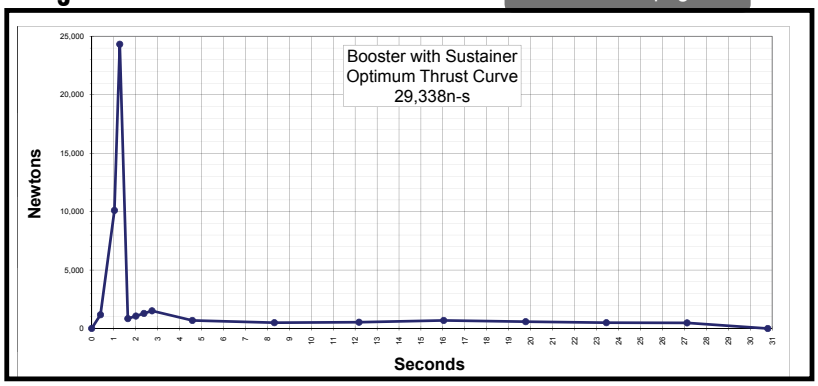

Table 5-3

Booster with Sustainer										4.5146E-04 kg/n-s		46.8 mass kg		n-s =	29,338
Alt	Op Vel	Delta Time	Elapsed Time	Loss Drag	Loss Grav	Optimum Delta V	Req'd Accel	Loss Impulse	Motor Impulse	Propellant Burned	Propellant Remaining	Rocket Mass	grams Burned	Motor Thrust	Accum
m	m/s	s	Sec	n - s	n - s	m/s/s	m/s/s	n - s	n - s	kg	kg	kg	per sec	n	n-s
0	0	0.00	0.0	0	0	0.00	0.0	0	0	0.00	13.245	46.8	0.0		
2	10	0.40	0.4	1	1	25.00	25.0	2	470	0.21	13.033	46.5	530.0	1,174	470
50	141	0.64	1.0	35	290	206.05	206.1	326	6,423	2.90	10.133	43.6	4560.9	10,102	6,892
100	272	0.24	1.3	70	104	541.03	541.0	174	5,891	2.66	7.474	43.9	10983.7	24,329	12,783
200	272	0.37	1.6	152	158	0.00	0.0	311	311	0.14	7.334	43.5	381.5	845	13,094
300	272	0.37	2.0	235	157	0.00	0.0	392	392	0.18	7.157	43.7	481.1	1,066	13,486
400	272	0.37	2.4	317	158	0.00	0.0	475	475	0.21	6.942	43.3	583.2	1,292	13,960
500	272	0.37	2.7	400	156	0.00	0.0	556	556	0.25	6.691	43.5	682.6	1,512	14,516
1,000	272	1.84	4.6	482	783	0.00	0.0	1,266	1,266	0.57	6.120	42.9	310.8	689	15,782
2,000	264	3.73	8.3	655	1,571	-2.28	-2.3	2,226	1,861	0.84	5.280	42.0	225.0	498	17,643
3,000	255	3.86	12.2	828	1,590	-2.20	-2.2	2,418	2,061	0.93	4.350	41.1	241.2	534	19,704
4,000	264	3.86	16.0	750	1,555	2.20	2.2	2,305	2,655	1.20	3.151	33.5	310.7	688	22,359
5,000	272	3.73	19.8	673	1,227	2.28	2.3	1,900	2,185	0.99	2.165	33.5	264.1	585	24,543
6,000	272	3.68	23.4	610	1,208	0.00	0.0	1,818	1,818	0.82	1.344	33.5	223.2	494	26,361
7,000	272	3.68	27.1	547	1,208	0.00	0.0	1,755	1,755	0.79	0.551	33.5	215.5	477	28,117
8,000	272	3.68	30.8	484	1,208	0.00	0.0	1,692	1,692	0.76	-0.213	33.5	207.8	460	29,809

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Continued from page 9

Keep in mind that the duration of each time increment will not be the same, and will have to be computed independently. Also, the calculations provide a mathematical approximation and increases in accuracy with smaller time intervals. I suggest creating another spreadsheet like **Table 5-3 (Page 9)** to make calculations for you.

Putting It Together

We finally have all the formulas to complete the table that will provide the required thrust for each time increment (thrust curve) that will yield the optimum velocity at each altitude as the rocket ascends. The **Table 5-3** column titled Motor Impulse (fifth column from the right) provides data points for the thrust curve. The **Table 5-3** column titled Elapsed Time (third from the left) provides the "seconds" data for the thrust curve. A motor with a similar thrust curve would provide the highest apogee for the rocket for a given motor size. **Figure 5-2 (Page 9)** provides the calculated optimum thrust curve for the sample rocket during booster flight.

We could find a commercial motor with a thrust curve closest to the optimum and use it for the flight, or design a custom motor with the specific thrust curve that is optimum for our rocket. The optimum thrust curve (**Figure 5-2**) doesn't look too exotic, a typical spike to get things moving, then a steady thrust to burnout. The original motor had an initial spike in the thrust curve, but only to just over 6000 newtons. If that is a limit, then the initial spike would need a longer duration to get things up to the "speed to fly" and to insure a stable launch.

RockSim It

As a check, I entered this designer thrust curve into the flight simulation program RockSim v8 and "flew" the sample rocket with the designer motor to compare with the simulation using the actual motor. The results of the simulation with the actual motor (**Figure 5-3**) and the designer motor (**Figure 5-4, Page 11**) indicate that the redesigned thrust curve will provide an altitude increase from the original 14,200 meters all the way to 28,700 meters! The prediction is for more than a 100% increase in the apogee altitude using the same size (total impulse) motor!

For the English units crowd, by utilizing the designed thrust curve, the predicted apogee altitude of the two stage rocket increased from 47,000 feet to 94,000 feet with no increase in the total motor impulse. There is still room for tweaking parameters like second stage ignition delay to get even more altitude.

With RockSim, you can enter your own motor thrust curve using the EngEdit program provided, and tweak the thrust curve as needed to achieve the highest altitude for your rocket. When you do this, keep in mind that the above spreadsheet calculations are very approximate for many reasons, and will only get you close to the optimum thrust curve. Several adjustments to the thrust curve may be required to get the most out of your total impulse. In addition, you may have to adjust the curve away from the optimum in order to have a motor that is possible to construct given motor design limitations.

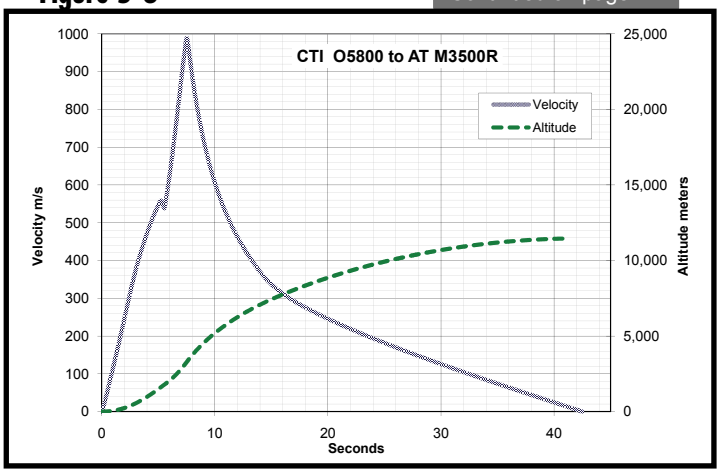
Chapter 6: Aft Closure

By now you are aware of the advantage of long burn motors and the subsequent need for a vertical trajectory for high altitude flights. When you read the failure analysis of many of the high altitude attempts, often the problem is tied to either aerodynamic failure due to extreme velocities necessary to achieve high altitude with an unguided flight, or recovery failure due to excessive horizontal velocities at apogee. Both of these problems can be mitigated if the rocket can be kept going straight up.

In the fifteen years that I have been working on the VTS system, the R/C hobby industry has increased the accuracy of the over-the-counter rate gyro systems. Now, Futaba has a rate gyro with "heading hold" that can partially bridge the

Figure 5-3

Continued on page 11



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gap between rate gyros and inertial guidance systems. Futaba also sells a Horizon Sensor they call a Pilot Assist Link, that could be used to control yaw and pitch for a vertical trajectory system. Parallax sells a Basic Stamp computer and devices such as a solid state compass I have used for roll control.

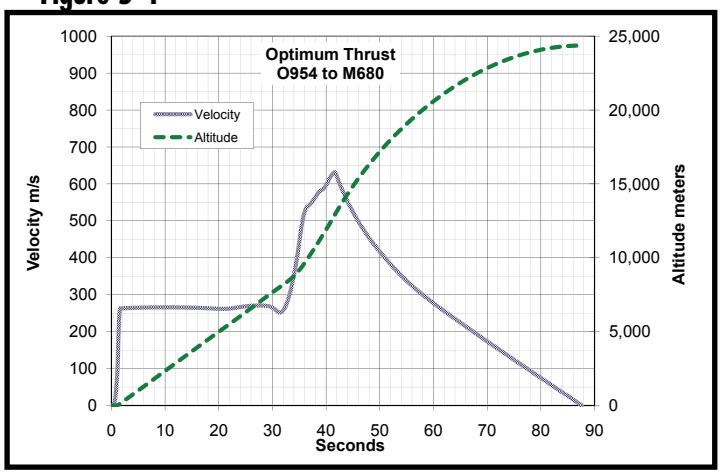
With the recent advances, it may be possible to use a long burn motor to achieve high altitude without excessive rocket velocities in the lower atmosphere. It may also be possible to keep the rocket oriented properly to allow the proper deployment of the recovery system. If this piques your interest in the advantages of a vertical flight profile, the next book to read is the book Vertical Trajectory Systems, also published by ARA Press.

About The Author:

Steve Ainsworth is a Civil Engineer with a Master's degree in Physics from UNLV. He has published articles in both HPR and ER since he became a "Born Again Rocketeer" in 1994. Steve joined Tripoli Vegas in 1995 and began flying his "gizmo" rockets. In 1995, Las Vegas was the perfect setting for re-entering rocketry with Gary Rosenfield of Aerotech providing motor tutoring and Tom Blazinin (TRA # 003) as the local Prefect, coaching and signing for Steve's Level 1 and Level 2 flights. While on an extended engineering assignment in northern California, Steve had evening time to think about rocketry. He used that time to learn enough aerodynamics to develop a computational method for flight predictions based on Gordon Mandell's MIT Thesis articles.

Download RockSim Files at: *www.ApogeeRockets.com/downloads/rockSim_files/Ainsworth.zip*

Figure 5-4



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