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## ***Finding Your Rocket's Altitude Using Single Station Tracking***

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## How to Perform Single-Station Altitude Tracking

By Norman Dziedzic Jr.  
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### Introduction

In model rocketry sooner or later, although usually sooner, you will come across the question, "How high did it go?" There are several ways to deal with this question such as:

1. Use the mfg.'s listed altitude.
2. Perform a computer simulation.
3. Include an altimeter within the model.
4. Track the model with a radar type system.
5. Visually track the model and use the observations to calculate the altitude.

Methods 1 and 2 give an approximate idea of altitude but cannot account for the actual conditions at the launch

Methods 3 and 4 can give accurate altitudes but are costly and the electronics required will not fit into smaller rocket bodies and add weight to the model. Also, using advanced radio methods may require special licenses.

Method 5 gives accurate results without adding any weight to the model and without requiring expensive components or radio operator licenses. In this article, we will investigate the visual tracking methods.

### Get on the Right Track

Visual tracking methods consist of measuring the line of sight from a fixed point or points on the ground to the model's apogee. The actual measurements taken are the angles from the horizontal (elevation) and in two point tracking, the angle in the plane of the ground (azimuth).

The angles are manipulated along with the known distances on the ground to determine the altitude the model reached. This process is sometimes called *data reduction*. Of course, to derive these procedures requires the use of some trigonometry—but fear not! You won't have to go through 11th grade math again. We'll just cover what's required for the altitude tracking and try to make it plain as pi.

### one is the Loneliest Number

The simpler but less accurate method of determining altitude is called Single Station Tracking (SST). As the name suggests, only one tracker is required. He or she stands a known distance from the launch pad and sights the flight of the model through a tracking device (we'll cover these instruments later). At apogee, the tracker fixes the position of the instrument and can then read off the angle of elevation (above horizontal) at which the apogee was observed as depicted in Figure 1.

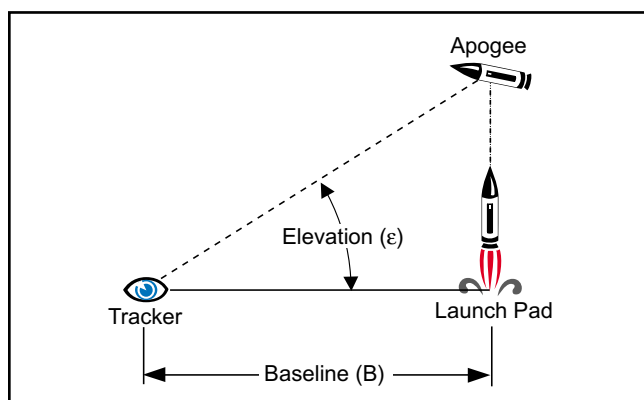


Figure 1: single station tracking

For SST to work, we have to assume that the model rocket will travel straight up. Since this is rarely the case, there can be quite a margin of error but SST is still better than "eyeballing" an altitude. Also, analyzing SST will give us a good footing to move on to Two Station Tracking (TST).

### Right as Rain - or - History repeats itself

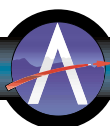
This straight up motion assumption leads to the generation of a 90° angle at the launch pad (see Fig. 1) between the rocket's flight path and the baseline. The triangle formed by those two lines and the tracker's line of sight is then called a right triangle and lets us make use of some powerful trigonometry to quickly and easily determine the altitude attained.

We will call the measured angle of elevation epsilon ( $\epsilon$ ). Angles are often labeled with Greek symbols to differentiate them from lengths which are usually labeled with regular letters but don't let that  $\epsilon$  scare you. It's just a name we give the

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## Single-Station Tracking

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angle until we make an actual measurement and can fill it in with a real value.

In our case, we know the length of B or the baseline because we had the tracker stand a known distance from the launch pad. What we want to find is A or the altitude so looking at Figure 2, we find that the third relationship using the tangent function has only one unknown quantity which is A. If we multiply both sides of the equation by the baseline length we come up with the equation to calculate the altitude from the measured angle and known baseline:

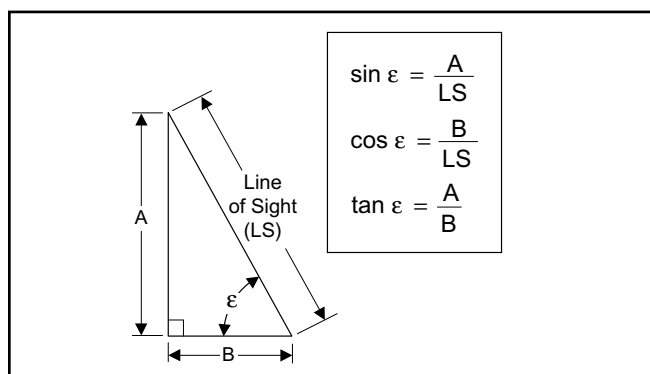
$$A = B \cdot \tan \epsilon \quad [1]$$

The tangent of the angle can be found using a simple hand held calculator. So, taking the example of a 150 ft baseline and a measured angle of elevation of 55° the altitude is given by  $150 \times \tan 55^\circ$  and the answer will have the units of ft. To perform this on your basic \$12 calculator just type in the following keys<sup>1</sup>:

150  $\times$  55  $\tan$  =

to find that the answer is about 214.2 ft.

You may have noticed that the units used for the baseline



**Figure 2: The right triangle and basic trigonometric relationships for right triangles**

are carried through to the calculated altitude. Thus, if your baseline is measured in meters, your calculated altitude will be in meters.

There are two other small parts to the altitude calculation. The tracker's eye is going to be some distance above ground level so this height can be added to the altitude. Similarly, the rocket as sitting on the launch pad is slightly above

ground level, so this reading should be subtracted from the altitude. For your own personal flight records, these matters are trivial but for competition flights, this may make a difference in the outcome of the event (note: as stated previously, single station tracking can have a large margin of error and thus is not allowed for official NAR sanctioned competitions).

### NO RUNS, NO HITS, SOME ERRORS

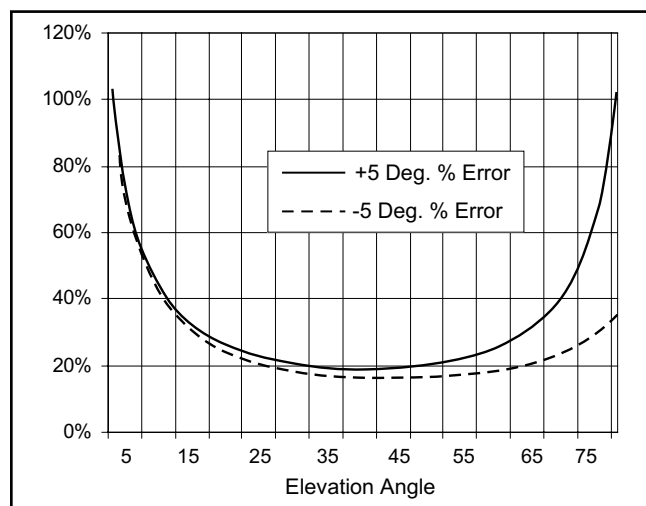
We have already mentioned that single station tracking is prone to errors but that's a very general statement. There are two major types of errors in SST. The first is tracker error in which the angle observed is not the true flight angle and the other is caused by a non vertical flight. Looking at how much each of these cases effects the calculated altitude may help us position the tracker to minimize the error for a given flying situation.

Let's look at the cases where a tracker measures an angle of 5° more and 5° less than the true angle. We will call these erroneous altitudes  $A^+$  and  $A^-$  and they can be calculated as:

$$A^+ = B \cdot \tan(\epsilon + 5^\circ) \quad [2]$$

$$A^- = B \cdot \tan(\epsilon - 5^\circ)$$

If we plot the two parts of equation [2] as a percent error from the true altitude (the one calculated from using just  $\epsilon$ ) we get Figure 3.



**Figure 3: Tracker Error Percent (+/-5 deg spotting error).**

There are two points of interest here. First is that the percentage of error is minimized right around a 45° elevation for both a plus and minus deviation from the real elevation. This shows us that it is best to set up the baseline for a flight to

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try to get an elevation reading of around 45°. Rearranging equation [1] gives:

$$B = \frac{A}{\tan(\epsilon)} \quad [3]$$

but for  $\epsilon=45^\circ$ ,  $\tan(\epsilon)$  is 1 so equation [3] reduces to  $B=A$ . In other words, to minimize the effects of tracker errors, set up your baseline to match your expected altitude. If this becomes difficult due to the expected altitude, you can use equ. [3] to set up a baseline to try to keep your expected elevation between 30° and 55°.

The second point Figure 3 shows is that the error in reading too large of an angle is always greater than that of reading too small of an angle. Therefore, trackers should pay special attention to avoiding overshoot in tracking.

### BACK TO BASICS

To move on, we have to cover a couple additional triangle/trigonometry basics. The first item is that for any triangle, the angles at the three corners will always add up to 180°. This means that whenever we know two of the angles of a triangle (say  $\alpha$  and  $\beta$ ), we can always calculate the third angle ( $\gamma$ ) by subtracting the other two from 180° ( $\gamma = 180 - \alpha - \beta$ ).

The other item is called the *Law of Sines* (LOS) and defines a relationship between the lengths of the sides of a triangle and the sines of the corner angles as shown in Figure 4.

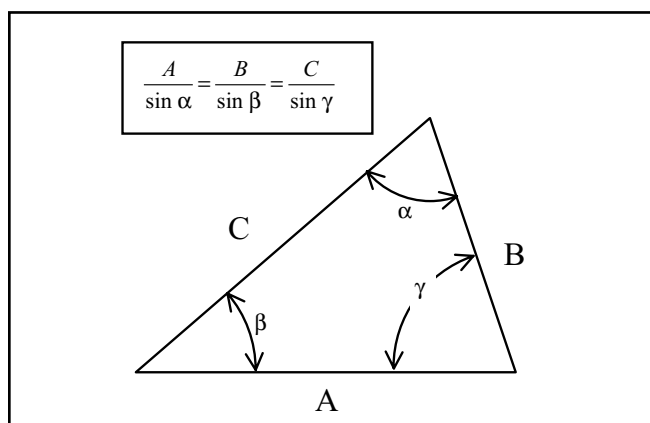


FIGURE 4: LAW OF SINES (LOS).

A, B, and C are the lengths of the sides of the triangle and  $\alpha$ ,  $\beta$ , and  $\gamma$  are the angles opposite those sides. The LOS applies to any triangle, not just right triangles.

### THE NOT SO STRAIGHT AND NARROW

We all know that no model rocket travels perfectly straight up. Using SST, any variation from this unattainable straight up path leads to errors in the calculated altitude. Let's look first at what happens when the model angles straight away from the tracker as shown in Figure 5. We'll call the deviations from vertical by the angle delta plus ( $\delta^+$ ).

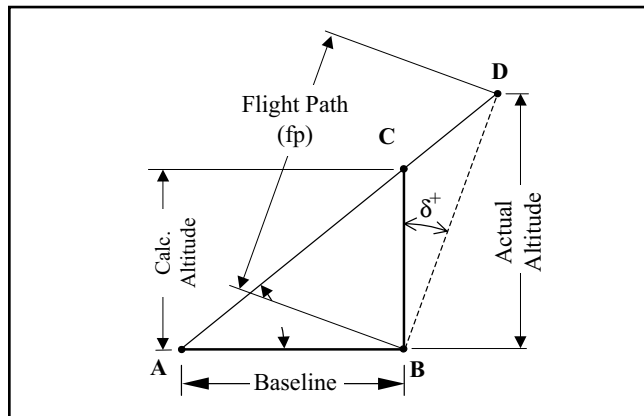


FIGURE 5: FLIGHT DEVIATION AWAY FROM TRACKER

In this case, the altitude calculated from equ. [1] will be less than the actual altitude attained. To find this real altitude, we'll first calculate the length of the flight path (fp). Looking at the triangle formed by the points ADB, the angle at point D is  $180 - \epsilon - (90 + \delta^+)$  or  $90 - \epsilon - \delta^+$ .

Then using the LOS, we can calculate fp from:

$$\frac{fp}{\sin \epsilon} = \frac{B}{\sin(90 - \epsilon - \delta^+)}$$

Or re-arranging:

$$fp = \frac{B \cdot \sin \epsilon}{\sin(90 - \epsilon - \delta^+)} \quad [4]$$

Then we can use the second right triangle relationship (Fig. 2) to get the actual altitude from fp:

$$\text{Altitude}^+ = fp \cdot \cos \delta^+ \quad [5]$$

Then combining equations [4] and [5] gives:

$$\text{Altitude}^+ = fp \cdot \frac{B \cdot \sin \epsilon \cdot \cos \delta^+}{\sin(90 - \epsilon - \delta^+)} \quad [6]$$

The condition when the model deviates toward the tracker is similar. In this case, the deviation angle is called  $\delta^-$  and the actual altitude is given by:

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$$\text{Altitude}^- = fp \frac{B \cdot \sin \epsilon \cdot \cos \delta^-}{\sin(90 - \epsilon - \delta^-)} \quad [7]$$

The last case we'll look at is when the model deviates to the left or right of the tracker. This situation is shown in Figure 6.

The math behind determining the true altitude for this case gets a little more complex and space is short so I'll just throw up the equation for the altitude

$$\text{Alt} = B \cdot \frac{\sqrt{1 - (\tan \epsilon \cdot \tan \delta)^2}}{2 - (\tan \epsilon \cdot \tan \delta)^2} \cdot \tan \epsilon \quad [8]$$

As we did before with the tracker error, Fig. 7 shows Equations 6, 7 and 8 plotted as a percentage error for the case where the deviation from vertical ( $\delta$ ) is  $10^\circ$  in the described direction.

Again, there are interesting things to be observed here. As with tracker errors, the greatest vertical deviation error is found when the model moves away from the tracker. Unlike tracker error, there is no minimum error point to these curves. The higher the elevation angle, the greater the effect of the error. But the real interesting point here is how much less a left/right deviation effects the error vs. the toward or away

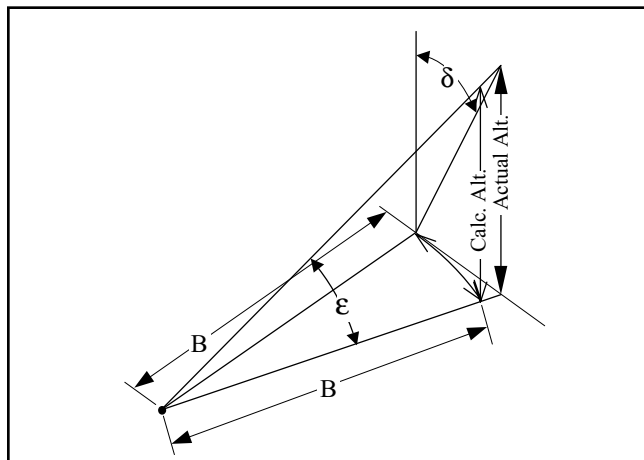


FIGURE 6: FLIGHT DEVIATION LEFT/RIGHT

cases.

### CONCLUSIONS

Single Station Tracking can be a simple and effective way to determine model rocket altitudes. The basic calculation used in SST and shown in equ. [1] was easily derived with a simple trigonometric relationship.

Looking further into equ. [1] we found that tracker errors are minimized when the baseline is chosen to get an elevation between  $30^\circ$  and  $55^\circ$ . Then digging deeper into the trig. and

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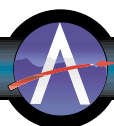
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with a little hand waving to get equ. [8] we saw that the direction a model deviates from vertical can make a large difference in the error of the altitude calculated.

We can summarize the findings in this article to give the following rules for Single Station Tracking:

1. Estimate your altitude and set up your baseline using equ. [3] with an angle  $\epsilon$  of between  $30^\circ$  and  $55^\circ$ .
2. Although trackers are always attempting to be exact, be especially careful to avoid overshooting the model as these errors are greater than undershooting.
3. Trackers should stand so that any prevailing winds will carry the model left or right as opposed to toward or away from them to minimize non-vertical flight path errors.

In the next issue, we'll take a look at Two Station Tracking (the method used in NAR Contests) and the physical devices used in tracking.

Questions, Comments or ideas for future articles can be

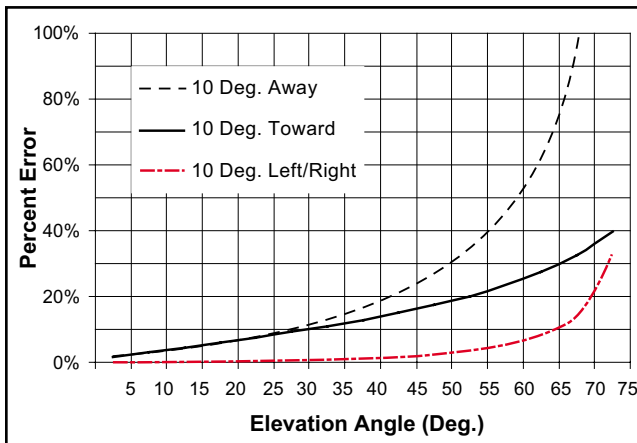


FIGURE 7: Non-vertical path errors

e-mailed to the author at [ndzied1@interaccess.com](mailto:ndzied1@interaccess.com)

### Footnotes:

<sup>1</sup> Hewlett Packard RPN type calculators and some other advanced calculators use a different order for pressing the keys. Consult your calculator manual if the given order doesn't work.

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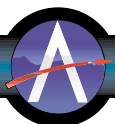
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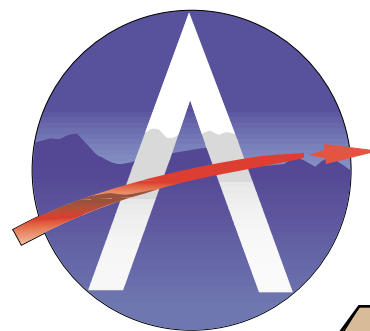


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