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How To Perform Two-Station Altitude Tracking

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How to Perform Two-Station Altitude Tracking

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The reason we are covering this topic is to help out those students and teachers participating in the NAR's Team America Rocketry Challenge. I recommend teams use this method of optical tracking for their practice flights. Because these rockets are going to travel so high, there is a good chance that the rocket might drift away. If you use the altimeter for your practice flights, and it floats away, you're going to lose your expensive payload. You should save the altimeter for the real flights that actually count.}

Introduction

In [Apogee E-zine Newsletter #92](#), we covered the single station tracking method for determining how high a rocket has flown. In this issue, we'll explore the two station tracking method.

This is the tracking method used by the National Association of Rocketry (N.A.R.) for determining altitudes in competition and in general is immune to the non-vertical flight errors which plague single station tracking.

Tea for Two

As the name suggests, two trackers are required for this tracking method. The two trackers are positioned a set dis-

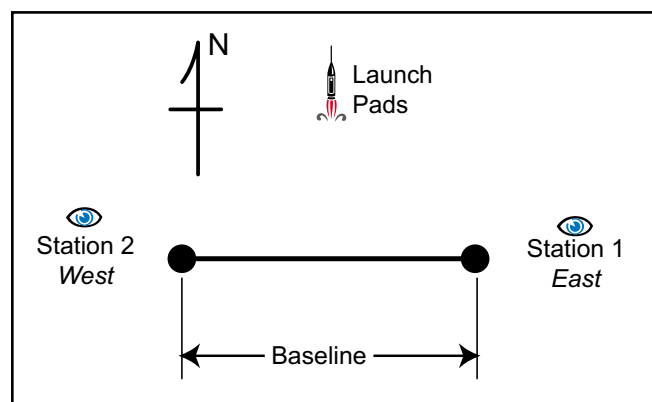


Figure 1: Tracker layout.

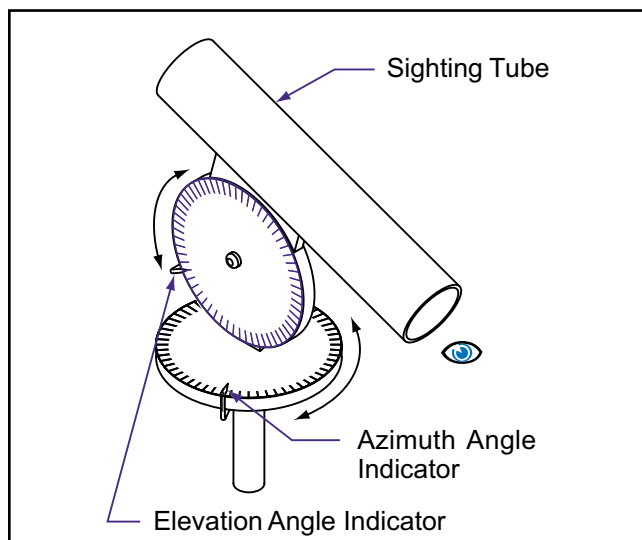


Figure 2: The Theodolite

tance apart on a baseline as seen in Figure 1. When the rocket is launched, the trackers follow the flight and zero in on the apogee using a device called a Theodolite (See Figure 2). This device allows the trackers to measure the angle in the plane of the ground and the angle of elevation at which they observed the model's apogee.

The layout of the baseline deserves a little discussion as it can make or break a tracking effort especially over the length of a day long launch. First, the baseline is spaced away from the launch area so that angles measured in the ground plane (azimuth angles) are not near zero. Referring back to the single station article, tracker errors were greatly magnified when elevation angles were near 0 or 90 degrees. The same is true for angles measured in two station tracking. If the baseline were in line with the launch area, flights which are close to straight up would generate almost no azimuth angle.

The baseline usually runs east/west and is offset north or south of the launch pad area to minimize the effect of the sun blinding the trackers. If the line was run north/south, flights veering east in the morning and west in the afternoon would be flying directly into the sun and be virtually impossible to track. Variations should be made to account for local condi-

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Archives of this Newsletter

All the articles that have appeared in this newsletter are archived at http://www.apogeerockets.com/education/newsletter_archive.asp

Two-Station Tracking

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tions include prevailing winds to avoid trackers looking into the sun.

BACK ON TRACK

So, the theodolites are located on either end of the baseline and set up so that the azimuth and elevation readouts point to zero when the theodolites are pointed directly at each other. Then the trackers follow the flight and lock in their theodolites at apogee or ejection (which is easier to see and synchronize between the trackers). The next step, called *data reduction*, is where the math comes into play to turn these 4 angle readings (and the known length of the baseline) into an altitude.

It's Greek to Me

Figure 3. shows the measured angles on a diagram of the tracking field. As touted, in the previous article, don't let the greek symbols bog you down, they are just there to hold a place for the angles until we actually take a reading. They

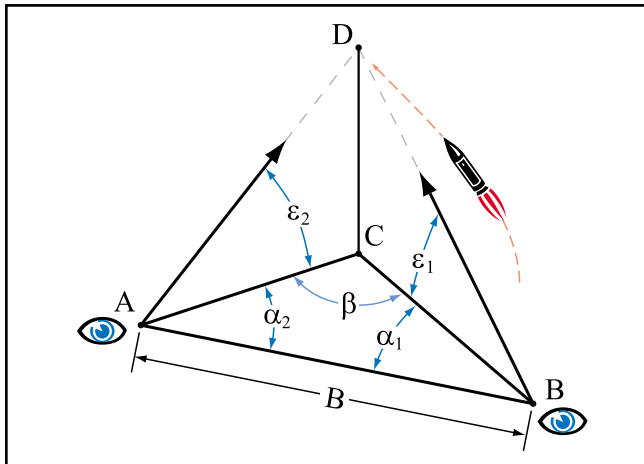


Figure 3: Triangulation Layout

also allow us to generate equations which can later be applied to any set of measurements we take.

The azimuth (or ground plane) angles are given by α_1 and α_2 (alpha one and alpha two) and the elevations by ϵ_1 and ϵ_2 (epsilon one and epsilon two). The angle at the back of triangle ABC (the triangle defined by traversing from point A to B to C and back to A) is given by β . Note that the labels east and west have been replaced by 1 and 2 to save space and make the equations easier to read.

Let's focus in on triangle ABC which lies in the ground

plane. First we'll calculate angle β from the fact that the sum of all of the angles for any triangle must add up to 180° . So angle β is given by:

$$\beta = 180 - (\alpha_1 + \alpha_2) \quad [1]$$

To move on, we need to find the lengths of the sides of triangle ABC and to do this we need to use another trick brought up in the single station tracking article: The Law of Sines as seen in Figure 4.

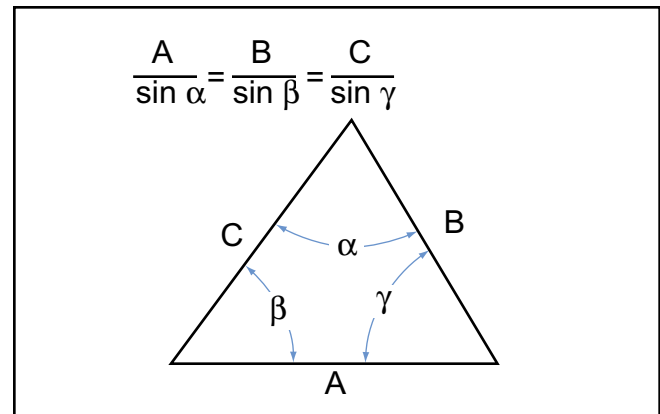


Figure 4: The Law of Sines

Remember, that we know the length of one side already; the baseline (B) which we set when we laid out the field. Also, from our tracking azimuth angles and equation [1] we know all of the angles.

Let's plug the length of side AC into the law of sines and re-arrange to get equation [2]:

$$\frac{AC}{\sin \alpha_1} = \frac{B}{\sin \beta}$$

$$AC = \frac{B \sin \alpha_1}{\sin \beta} \quad [2]$$

Similarly, we can find the length of side BC as follows:

$$BC = \frac{B \sin \alpha_2}{\sin \beta} \quad [3]$$

Now let's look at the angle formed by the points BCD as seen shaded in Figure 5.

The corner of this triangle at point C is a 90 degree corner making this a *right triangle*. Inspecting this triangle we see that the length from C to D is the altitude we seek and the

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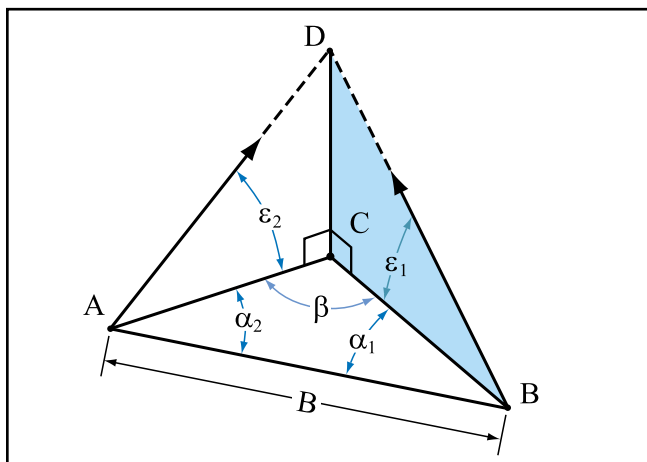


Figure 5: Triangle BCD

length from B to C is the one we just calculated. The ratio of CD to BC is a definition for the tangent trig function for angle ϵ_1 . So:

$$\tan(\epsilon_1) = \frac{CD}{BC}$$

Or... re-arranging gives:

$$CD = \tan(\epsilon_1) \cdot BC$$

and calling CD by Alt_1 yields:

$$Alt_1 = \tan(\epsilon_1) \cdot BC \quad [4]$$

Perfectly Average

In a perfect world this would be the end of our calculations since we have come up with an equation for the altitude. However, there is almost certainly some discrepancy between the readings taken by the two trackers.

To address this difference, triangle ACD is inspected in a manner similar to triangle BCD. This leads to a second equation for altitude:

$$Alt_2 = \tan(\epsilon_2) \cdot AC \quad [5]$$

In order to come up with a final altitude, the altitudes computed in equations [4] and [5] are averaged:

$$Altitude = \frac{Alt_1 + Alt_2}{2} \quad [6]$$

case closed?

So we have an altitude but what if the two component altitudes differ greatly? How do you know if you can trust the average value?

An accepted procedure is to calculate the difference between the two readings as a percentage.

$$Err = \frac{Alt_1 - Alt_2}{Alt_1 + Alt_2} \cdot 100\% \quad [7]$$

Then if the this difference is less than or equal to 10% the averaged altitude is accepted as the actual altitude. If the error exceeds 10% the situation is referred to as a *Track Not Closed*.

The procedure just outlined is called the *Vertical Mid-Point Data Reduction* method. In it we first assumed the two tracker line of sights intersected. Then we used trigonometry along with the triangles formed by the tracking lines and baseline to determine the altitude.

Another form of data reduction is called the *Geodesic* method. This method starts by assuming that the tracker line of sights do not intersect. Then the shortest line connecting these skew lines is found with the calculated altitude being at the mid point of this line. This involves vector math and will probably end up the subject of it's own article.

The Big Square Dance

At large competitions such as NARAM where the number of flights is great and the number of trackers is plentiful (due to mandatory range duty for competitors) you can use more trackers to improve the chances of getting a closed track.

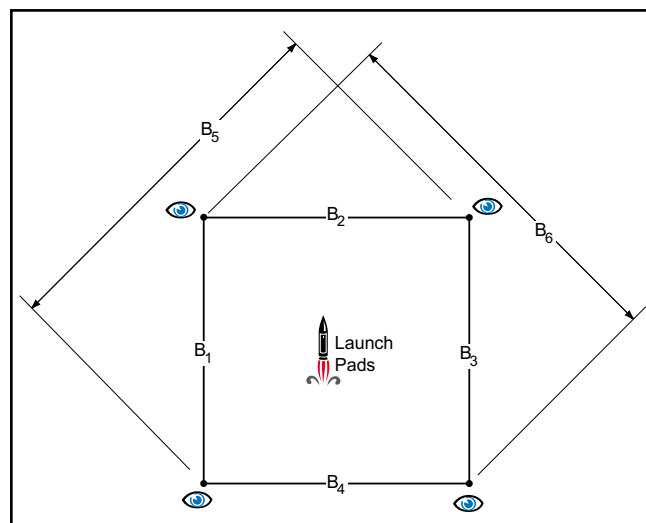


Figure 6: The Tracking Square

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Figure 6 shows a layout of 4 trackers in a square around the launch pads. In this setup, there are 6 possible baselines to use for calculating the altitude: the four sides of the square and the two diagonals.

All 6 altitudes are calculated and those that have an error of less than or equal to 10% are averaged to determine the actual altitude.

check - double check

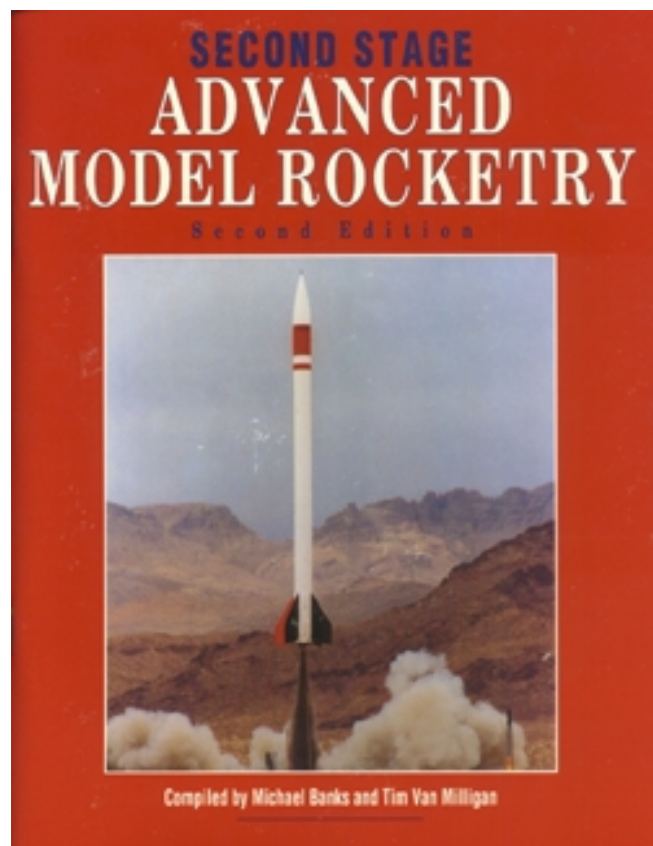
If you want to perform some of your own calculations, here are a couple of examples for you to use to double check

α_1	ϵ_1	α_2	ϵ_2	Alt	Err
50	35	70	40	225	1.2%
66	41	30	32	152	13.5%

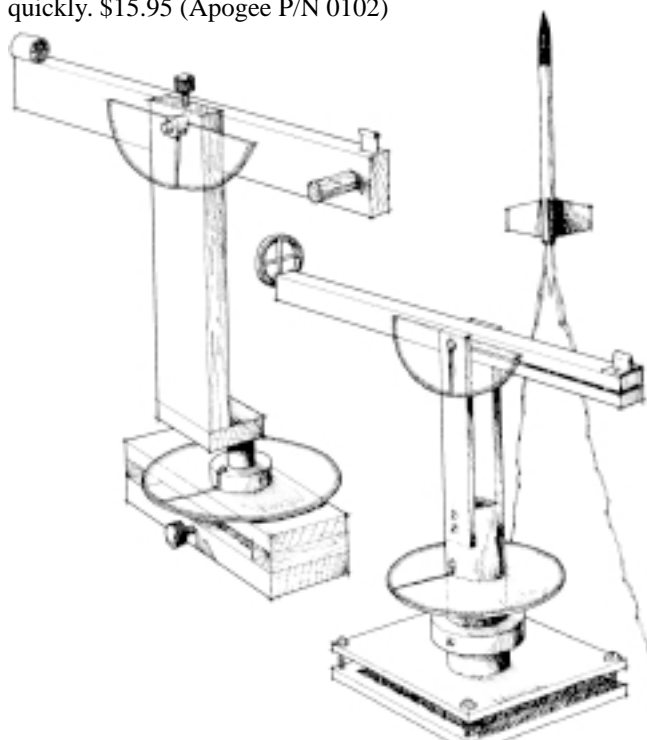
your work (Based on a 300 m baseline).

So, now you know how actual altitudes are calculated in competitions. If you want to try to track your own models, you best bet is to contact a club in your area to see if they have a set of theodolites which you can borrow. With a little practice you can hone in on your models actual altitudes gain new insights into your own Rocket Science.

Tracking Scope Construction Plans



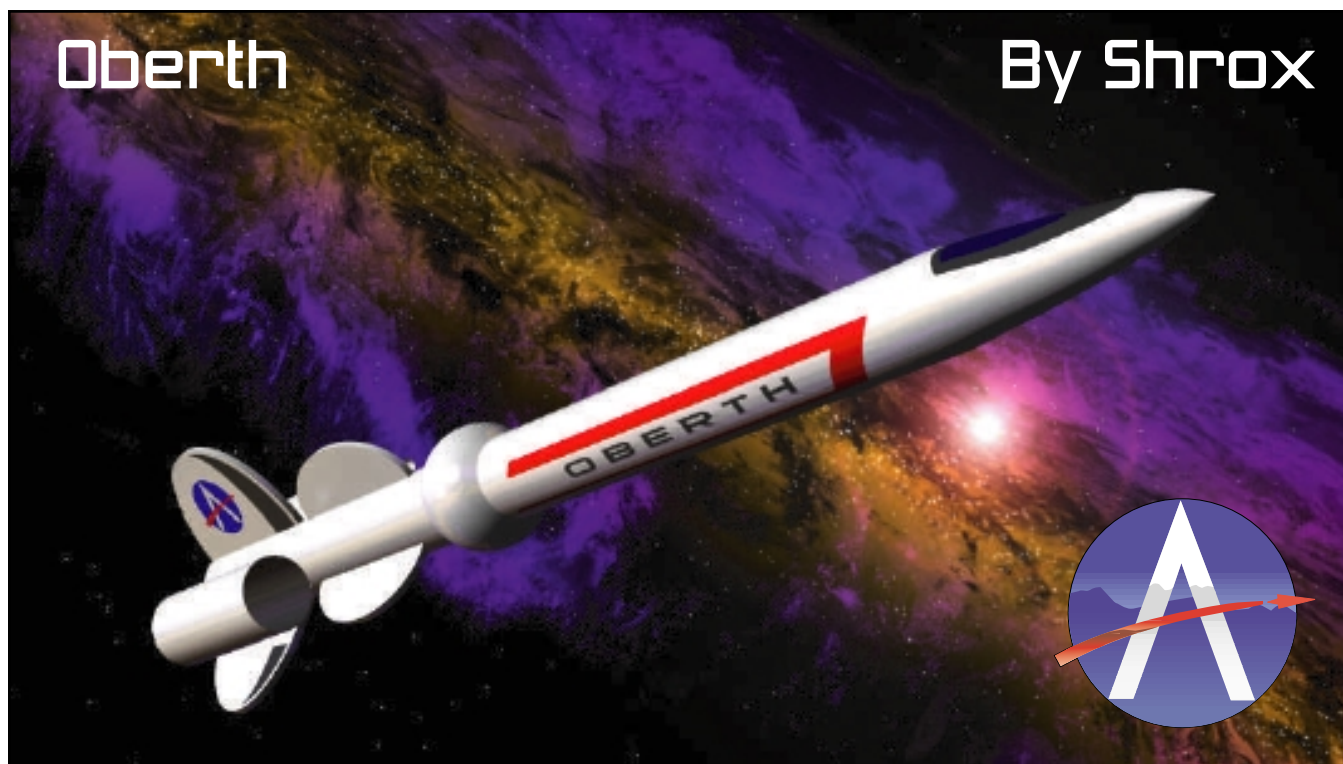
In the book "Second Stage: Advanced Model Rocketry," You'll find plans for two different theodolite style tracking scopes! These can be used to track how high your rockets fly. Our supplies of this book are limited, so be sure to order quickly. \$15.95 (Apogee P/N 0102)



For more information, visit our web site at: http://www.apogeerockets.com/2nd_stage_book.asp

Oberth

By Shrox



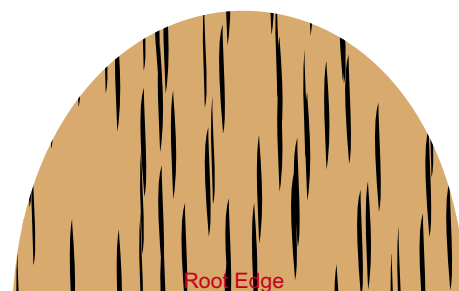
*Download the RockSim Plans
And Decal Artwork!*

The data file you need is at:
<http://www.ApogeeRockets.com/shrox/oberth.html>



Parts List:

- 1 - Nose cone. Apogee P/N 19400 - PNC-24A
 - 1 - Body tube. Apogee P/N 10099 - 24mm dia X 13.5 inches long.
 - 2 - Body tube. Apogee P/N 10099 - 29mm dia X 2.5 inches long.
 - 1 - Body tube. Apogee P/N 10085 - 18mm dia X 2.75 inches long.
 - 1 - Motor block. Apogee P/N 13031 - CR 18/24
 - 1 - Apogee Streamer P/N 29005 - 48 inches long X 2 inches wide.
 - 1 - Shock cord. Apogee P/N 29505 - 100lb test Kevlar® X 36 inches long)
 - 1 - Shock cord mount.
 - 1 - Ping Pong ball.
 - 1 - Launch lug. Apogee P/N 13051 1/8 inch X 1 inch long.
- Fin Material - Basswood or Balsa wood, 3/32 inch thick.



Make 3 of each size

